

# The Effects of Macroeconomic Policy on the Long-Term and Short-Term Interest Rates<sup>†</sup>

Toru Nagahara<sup>\*</sup>

## ABSTRACT

In this paper, we examine the effects of macroeconomic policy on the long- and short-term rates in the system in terms of the market equilibrium analysis. We assume that the nominal yield on two-period bonds is an average of the current short-term rate and the subsequent short-term rate plus the increasing risk premium with respect to expected fluctuations of prices. We also assume the expectations of inflation (deflation) are endogenous and are rationally determined in the goods market. Under this setting, the process of generating risk premium greatly influences the efficacy of macroeconomic policy, in the sense that the policy does not swing both the rates, the long-term rates in particular. The paper, therefore, implies that policy makers should pay as close attention as possible to implementing the policy so that it becomes an effective measure. In other words, this study demonstrates that the accountability of the policy is important.

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† This paper is an extended version of the presentation which was delivered at the annual meeting of Japan Economic Policy Association held at Kwansei Gakuin University, May 30, 2004. I am grateful for the helpful comments and suggestions from Yukitoshi YAMADA, who was my discussant, and two anonymous referees. Any errors are the responsibility of my own.

\* Graduate School of Economics, Rikkyo University E-mail: [tnlennon@poem.ocn.ne.jp](mailto:tnlennon@poem.ocn.ne.jp)

## 1 INTRODUCTION

The Bank of Japan (BoJ) has implemented the quantitative monetary easing. In other words, the BoJ commits itself to maintaining short-term rates at low levels for several years, and tries to achieve stabilization of long-term rates at low levels, which referred to as the so-called “policy duration effect”. It seems that arguments<sup>1)</sup> on this effect, when traced, lead to the expectations hypothesis about the term structure of interest rates. Now we will establish certain expectations hypotheses as well as introduce their research genealogy up to the present.

According to Shiller (1990), the earliest argument of the term structure emerged at the end of 19th century when it was not treated academically at that time<sup>2)</sup>. Subsequently, Lutz (1940) argued the pure expectations hypothesis<sup>3)</sup> that the current  $n$ -period long-term rate equals an average of the current short-term rate and the future short-term rates until the  $(n-1)$ -period. The formulation is given by

$$(1) \quad R_t^n = \frac{r_t + E_t r_{t+1} + \cdots + E_t r_{t+n-1}}{n},$$

where  $R_t^n$  is the  $n$ -period long-term rate at time  $t$ ,  $r_{t+j}$  ( $j = 0, 1, \dots, n-1$ ) is the one-period short-term rate at time  $t+j$ , and  $E_t$  is the expectation formed at time  $t$ . In contrast, the hypothesis involving risks specific to asset management in the long run was presented by Hicks (1946). This adds a risk premium proportional to a maturity date to equation (1) by Lutz (1940), and is represented by

$$(2) \quad \tilde{R}_t^n = \frac{r_t + E_t r_{t+1} + \cdots + E_t r_{t+n-1}}{n} + \phi_t,$$

where  $\phi_t$  is the risk premium at time  $t$  which increases as the maturity  $n$  increases. When the risk premium  $\phi_t$  is zero,  $\tilde{R}_t^n$  in equation (2) corresponds to  $R_t^n$  in (1). Hence, we can regard equation (1) as a special case of equation (2). Accordingly, the formulation by equation (2) can be referred to as the general expectations hypothesis.

While the risk premium of the general hypothesis was regarded as proportional to maturity dates in Hicks (1946), there also exists the viewpoint that it varies with

1) See, for example, Okina and Shiratsuka (2003) and Nagayasu (2004).

2) Shiller (1990, p. 644).

3) Refer to Shikano (1984), with regard to the hypotheses given below.

time. This study considers both the theory of the premium being calculated by a statistical method as described in Shiller (1990), and the preferred habitat theory in which the premium is determined in accordance with an investor's preference for a maturity date in Modigliani and Sutch (1966). Taking this into consideration, the premium being invariant over time as presented by Hicks (1946) is nothing but a special example of the time-varying premium. The hypothesis involving the latter premium, therefore, may be referred to as the more general hypothesis.

In addition to these theoretical analyses, although a number of empirical analyses have been performed, Fukuta (1991) indicates that their results are generally inconsistent with the theoretical hypothesis<sup>4)</sup>. However, it seems that this is because they assume a constant risk premium. In fact, the conclusion in Shirakawa (1987), where the term structure is analyzed in terms of consumption-based capital asset pricing model (CCAPM) involving the time-varying risk premium, is consistent with the expectations theory<sup>5)</sup>.

In this study we examine the expectations hypothesis with the risk premium that increases with respect to the uncertainty in the Knight (1921)'s sense<sup>6)</sup>, based on the general equilibrium analysis. Further, we derive certain implications for the current Japanese economy by focusing on the trend of long-term rates by the revival of business or the fear for a gap between demand and supply of long-term bonds, which would be caused by postal privatization. In contrast with the term structure that uses the arbitrage pricing model (APM) under the intertemporal general equilibrium system by Cox, Ingersoll, and Ross (1985), it is important to note that this paper aims at analyzing the term structure of interest rates in terms of the general equilibrium approach to monetary market by Tobin (1969). The remaining paper is organized as follows. Section 2 explains the model that forms the basis of this paper. We specify the behavior of economic agents in our model, on the basis of Horiuchi (1980) in Section 3. Section 4 examines the endogenously determined expected prices, which are crucial parameters in this model, based on rational expectations. On the basis of these sections, we can build the market general equilibrium system in our

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4) Fukuta (1991, p. 113).

5) Fukuta (1991, p. 125-6).

6) Whether prices are inflationary or deflationary, as shown in equation (4) below, the risk premium increases with respect to the expected fluctuations of prices. It is in this sense that the terms of the uncertainty in the Knight (1921)'s sense are used.

model, and then discuss the effects of changes in exogenous variables on the short-term and long-term rates, using comparative statics, in Section 5. In Section 6, we conclude the study by drawing a comparison between our results and the conclusions from the expectations hypothesis introduced by Romer (1996, ch. 9), and highlighting the issues in the latter and our future challenges.

## 2 BASIC MODEL

### 2.1 Analytical Framework

The following is the analytical framework presented in this paper. The economy is inhabited by five types of agents: commercial banks, firms, households, the central bank, and financial authorities; the notations are  $b$ ,  $f$ ,  $h$ ,  $cb$ , and  $fa$  respectively. As discussed in the following section, assume that commercial banks act on a profit maximization principle and that each of the others behaves according to *ad hoc* suppositions.

Meanwhile suppose that there are five types of financial assets, that is, loans ( $L$ ), bonds ( $B$ ), call money ( $C$ ), deposits ( $D$ ), and borrowed reserves ( $\overline{BR}$ )<sup>7)</sup>, and four types of markets—the loans market, the bonds market, call market, and the deposits market—for each of the four assets except  $\overline{BR}$ . Furthermore, there are four types of rates—the loan rate ( $r_L$ ), the bond rate ( $R$ )<sup>8)</sup>, the call rate ( $r_C$ ), and the deposit rate ( $r_D$ ) respectively—in these markets. While the first three are endogenously determined in the system to adjust their market disequilibrium, the last is fixed at an exogenously given level. On the other hand, we suppose the goods market to be a real market in this economy, which we will examine in detail below. Further, let the long- and short-term rates in question substitute with the bond rate,  $R$ , and call rate,  $r_C$ , respectively.

Finally note the following dynamics of our model. Suppose the two-period model, following the examples of Mankiw and Miron (1986) and Walsh (2003), which consists of the present and the future, and let the long rate be a maturity 2 and the

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7) For simplicity, let  $\overline{BR}$  be constant. See Nagahara (2003, n. 4) for a confirmation of the validity of this assumption.

8) As will be examined in Section 3, it is precisely  $E_t r_{C, t+1}$  that plays a role in adjusting the disequilibrium in the bonds market. The bond rate changes as a result of a change in  $E_t r_{C, t+1}$ .

short rate be maturity 1. With regard to the former rate, we consider the following formulation so that our two-period model is consistent with equation (2): The long-term rate is equal to an average of the current short-term rate and the subsequent short-term rate plus the risk premium, which we will mention below.

## 2.2 Premise of Each Market

As stated above, we have four types of financial asset markets and a real market. With regard to financial markets, assume that the three markets, except for the deposits market, are competitive<sup>9)</sup>. On the other hand, we assume that the deposits market is always balanced at a given time because commercial banks supply deposits in response to the (household's) deposit demand being determined at a given deposit rate.

In the goods market, which is the only real market in this paper, supply and demand for a representative good are adjusted by the inflation rate. In other words, the disequilibrium existing at time  $t$  is adjusted by the inflation rate for the same period  $\pi_t$ . We assume that the inflation rate, which is expected at time  $t$ , to prevail at  $t+1$ ,  $E_t\pi_{t+1} = \pi_{t+1}^e$ , is determined rationally, which will be examined in Section 4.

## 2.3 Definition of the Long-Term Rate

In our two-period model, the long-term rate is expressed by the numerical form

$$(3) \quad R_t = \frac{r_{C,t} + E_t r_{C,t+1}}{2} + \phi_t,$$

where  $\phi_t$  is a function that represents the risk premium and a subscript of each variable,  $t$ , is a point in time. The risk premium function  $\phi_t$  takes various forms according to based hypotheses. For example, the risk premium from Hicks (1946) is a linear function whose independent variables are maturity periods. As stated above, we consider an increasing risk premium due to the uncertainty of future prices. Then, following Shikano (1984)<sup>10)</sup>, we assume the following premium. The more

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9) As Stiglitz and Greenwald (2003, p. 2) state, it may be more appropriate not to "think of the market for loans as identical to the market for ordinary commodities." This study has limitations in this sense because it regards the loans market to be similar to the commodities market. Therefore, a close investigation of the credit market must be addressed in future works.

10) See Shikano (1984, p. 47).

difficult the forecast of prices, the larger is the premium.

$$(4) \quad \phi_t = \theta(\pi_{t+1}^e)^2, \theta \geq 0,$$

where  $\pi_{t+1}^e$  is the expected rate of inflation, and  $\theta$  is a parameter that represents the impact of price fluctuations on the risk premium. It implies that, when  $\theta$  is large, the impacts are enormous, i.e., the extent of investor's surprise is high, while he/she is neutral to the fluctuations in case of  $\theta = 0$ . Equation (4) shows that the premium becomes large irrespective of whether investors anticipate inflation or deflation. However, note that a first-order derivative with respect to the expected inflation,  $\phi'_t = 2\theta\pi_{t+1}^e$ , is

$$(5) \quad \phi'_t = \begin{cases} 2\theta\pi_{t+1}^e > 0 & \text{if } \pi_{t+1}^e > 0, \\ 2\theta\pi_{t+1}^e < 0 & \text{if } \pi_{t+1}^e < 0. \end{cases}$$

In brief, the derivative has the opposite sign depending on whether the price expectations are inflationary or deflationary.

### 3 EACH AGENT'S BEHAVIOR

In this section, we specify each behavior of the five agents in our model. Let us define the superscripts or subscripts that appear in the mathematical manipulations mentioned below: Firstly, let the superscripts represent agents in this economy; secondly, denote a subscript  $t$  as a point in time; and thirdly, let the other subscripts that are attached to the operators that indicate a function be notations with regard to partial differential. For example,  $A_{L,t}^b \equiv \partial A_t^b(L_t^b, B_t^b) / \partial L_t^b$ ,  $A_{BL,t}^b \equiv \partial^2 A_t^b(L_t^b, B_t^b) / \partial B_t^b \partial L_t^b$ ,  $L_{L,t}^b \equiv \partial L_t^b(r_{L,t}, R_t, r_{C,t}) / \partial r_{L,t}$ , and so on.

#### 3.1 Commercial banks

Portfolio management of a representative commercial bank is defined as the profit maximization problem

$$(6) \quad \text{Max.} \quad \Pi_t^b = r_{L,t} L_t^b + R_t B_t^b - r_{C,t} C_t^b - r_{D,t} D_t^b - r_{BR,t} \overline{BR}_t - A_t^b(L_t^b, B_t^b),$$

$$(7) \quad \text{s.t.} \quad kD_t^b + L_t^b + B_t^b = D_t^b + \overline{BR}_t + C_t^b,$$

$$(8) \quad R_t = \frac{r_{C,t} + E_t r_{C,t+1}}{2} + \theta (\pi_{t+1}^e)^2,$$

where  $r_{BR,t}$  is the discount rate,  $A_t^b(L_t^b, B_t^b)$  is the adjustment cost function of the bank, and  $k$  is the required reserve ratio. Equation (6) shows that the current bank's profits consist of the operating income gained from portfolio management minus the

sum of fund costs and other costs. Assume that  $A_t^b$  is convex and twice continuously differentiable<sup>11</sup>). Equation (7) is defined from the bank's balance-sheet. Since  $C_t^b$  is on the credit side, it is supposed here that the bank always demands call money. In other words, we suppose the call demand by banks that are short of money exceeds the supply by banks holding surplus money, and that excess demand as a whole is accommodated through operations to the call market by the central bank. Equation (8), as discussed in the previous section, represents that the long-term rate consists of the average of the current and future short-term rates at the first term in (8) plus the risk premium at the second one. It is important that the long-term rate itself is determined depending on each endogenous variable in the system. Thus, given that the call rate,  $r_c$ , and the expected rate of inflation,  $\pi_{t+1}^e$ , are decided in the call market and the goods market respectively, it is  $E_t r_{c,t+1}$  that plays a role in adjusting the gap between demand and supply in the bond market<sup>12</sup>).

Substituting (7) for (6), we have

$$(9) \quad \begin{aligned} \Pi_t^b = & (r_{L,t} - r_{c,t}) L_t^b + (R_t - r_{c,t}) B_t^b \\ & + [(1-k) r_{c,t} - r_{D,t}] D_t^b + (r_{c,t} - r_{BR,t}) \overline{BR}_t + A_t^b(L_t^b, B_t^b). \end{aligned}$$

Equation (9) shows that the bank's profit increases considerably when the discount rate,  $r_{BR,t}$ , or the required reserve ratio,  $k$ , is lower. The first-order conditions to maximize profit are given by

$$(10) \quad \partial \Pi_t^b / \partial L_t^b = r_{L,t} - r_{c,t} - A_{L,t}^b(L_t^b, B_t^b) = 0,$$

$$(11) \quad \partial \Pi_t^b / \partial B_t^b = R_t - r_{c,t} - A_{B,t}^b(L_t^b, B_t^b) = 0.$$

Conditions (10) and (11) indicate that the marginal profit of each earning asset is equal to fund costs plus the marginal costs from investment in each asset.

Differentiating equations (10) and (11) with respect to  $r_{L,t}$ , they can be rewritten as

$$(12) \quad \begin{aligned} 1 = & A_{LL,t}^b \frac{dL_t^b}{dr_{L,t}} + A_{LB,t}^b \frac{dB_t^b}{dr_{L,t}}, \\ 0 = & A_{BL,t}^b \frac{dL_t^b}{dr_{L,t}} + A_{BB,t}^b \frac{dB_t^b}{dr_{L,t}}, \end{aligned}$$

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11) For details, see equation (13).

12) With respect to the expected short-term rate, it is often used in empirical analyses as a dependent variable. See equation (4) in Mankiw and Miron (1986, p. 214) and equation (16) in Mankiw (1986, p. 77).

$$\Leftrightarrow \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} A_{LL,t}^b & A_{LB,t}^b \\ A_{BL,t}^b & A_{BB,t}^b \end{bmatrix} \begin{bmatrix} dL_t^b / dr_{L,t} \\ dB_t^b / dr_{L,t} \end{bmatrix}$$

Let  $H_t^b$  be a  $(2 \times 2)$  matrix in the right side of equation (12).  $|H_t^b| > 0$  since  $A_t^b(\cdot)$  is a convex function. With respect to the Hessian elements, the following is assumed

$$(13) \quad \frac{\partial^2 A_t^b}{(\partial L_t^b)^2}, \frac{\partial^2 A_t^b}{(\partial B_t^b)^2} > \frac{\partial^2 A_t^b}{\partial L_t^b \partial B_t^b} = \frac{\partial^2 A_t^b}{\partial B_t^b \partial L_t^b} = 0.$$

Equation (13) implies that costs such as monitoring cost and labor cost gradually increase as the management of assets is increased. On the other hand, the cross-effects are nil when economies of scope exist in the bank's management of assets as in Freixas and Rochet (1997, pp. 54-5). Consequently, we assume  $A_{LB,t}^b = A_{BL,t}^b = 0$  for simplicity.

Thus, we can solve equation (12) using assumption (13) and  $|H_t^b| > 0$  as follows:

$$\frac{dL_t^b}{dr_{L,t}} = \frac{1}{|H_t^b|} \times \begin{vmatrix} 1 & 0 \\ 0 & A_{BB,t}^b \end{vmatrix} = \frac{A_{BB,t}^b}{|H_t^b|} > 0,$$

$$\frac{dB_t^b}{dr_{L,t}} = \frac{1}{|H_t^b|} \times \begin{vmatrix} A_{LL,t}^b & 1 \\ 0 & 0 \end{vmatrix} = 0.$$

Then inserting (8) into (11), we obtain

$$(11') \quad \frac{E_t r_{C,t+1} - r_{C,t}}{2} + \theta (\pi_{t+1}^e)^2 = A_{B,t}^b(L_t^b, B_t^b).$$

Differentiating equations (10) and (11') with respect to,  $E_t r_{C,t+1}$ ,  $r_{C,t}$ , and  $\pi_{t+1}^e$  respectively yields

$$(14) \quad \begin{bmatrix} 0 \\ \frac{1}{2} \end{bmatrix} = \begin{bmatrix} A_{LL,t}^b & 0 \\ 0 & A_{BB,t}^b \end{bmatrix} \begin{bmatrix} dL_t^b / d(E_t r_{C,t+1}) \\ dB_t^b / d(E_t r_{C,t+1}) \end{bmatrix},$$

$$(15) \quad \begin{bmatrix} -1 \\ -\frac{1}{2} \end{bmatrix} = \begin{bmatrix} A_{LL,t}^b & 0 \\ 0 & A_{BB,t}^b \end{bmatrix} \begin{bmatrix} dL_t^b / dr_{C,t} \\ dB_t^b / dr_{C,t} \end{bmatrix},$$

$$(16) \quad \begin{bmatrix} 0 \\ \phi_t' \end{bmatrix} = \begin{bmatrix} A_{LL,t}^b & 0 \\ 0 & A_{BB,t}^b \end{bmatrix} \begin{bmatrix} dL_t^b / d\pi_{t+1}^e \\ dB_t^b / d\pi_{t+1}^e \end{bmatrix}.$$

Solving (14) (16) as mentioned above, the behavior of the bank is given by<sup>13)</sup>

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13) These calculations are not shown here since they are analogous to the solution for



$$(17) \quad L_t^b = L_t^b(r_{L,t}, E_t r_{C,t+1}, r_{C,t}; \pi_{t+1}^e), L_{L,t}^b > 0, L_{Er,t}^b = 0, L_{C,t}^b < 0, L_{\pi,t}^b = 0,$$

$$(18) \quad B_t^b = B_t^b(r_{L,t}, E_t r_{C,t+1}, r_{C,t}; \pi_{t+1}^e), B_{L,t}^b = 0, B_{Er,t}^b > 0, B_{C,t}^b < 0, \\ B_{\pi,t}^b > 0 \text{ (if } \pi_{t+1}^e > 0), B_{\pi,t}^b < 0 \text{ (if } \pi_{t+1}^e < 0).$$

When the loan rate rises (falls), the investment in loans is increased and the investment in bonds remains unchanged, while the investment in bonds is increased and the investment in loans remains constant in case of a rise (fall) of the expected short-term that leads to a rise (fall) of the bond rate. The rising (falling) call rate reduces (increases) both assets due to increasing (decreasing) costs. Finally, the expected fluctuations of prices raise the risk premium, and, as a result, expand the spread between the long-term rate and the short-term rate since the larger premium raises the long-term rate but not the short-term rate. Consequently, the bank does not change the share of loans but increases the allocation of bonds.

Substituting equations (17) and (18) into equation (7), we have

$$(19) \quad C_t^b \equiv L_t^b(r_{L,t}, E_t r_{C,t+1}, r_{C,t}; \pi_{t+1}^e) + B_t^b(r_{L,t}, E_t r_{C,t+1}, r_{C,t}; \pi_{t+1}^e) \\ - (1-k) D_t^b(r_{D,t}) - \overline{BR}_t.$$

Totally differentiating (19) with respect to  $C_t^b$ ,  $r_{L,t}$ ,  $E_t r_{C,t+1}$ ,  $r_{C,t}$ , and  $\pi_{t+1}^e$  yields

$$(20) \quad dC_t^b = \underbrace{[L_{L,t}^b + B_{L,t}^b]}_{(+)} dr_{L,t} + \underbrace{[L_{Er,t}^b + B_{Er,t}^b]}_{(0)} d(E_t r_{C,t+1}) \\ + \underbrace{[L_{C,t}^b + B_{C,t}^b]}_{(-)} dr_{C,t} + \underbrace{[L_{\pi,t}^b + B_{\pi,t}^b]}_{(0) \text{ (+) or (-)}} d\pi_{t+1}^e.$$

The condition for the first expression in brackets on the right-hand side of equation (20) is  $L_{L,t}^b (\equiv A_{BB,t}^b / |H_t^b|) + B_{L,t}^b (\equiv 0) > 0$ . Similarly, the conditions for the second, third, and fourth expressions in bracket are as follows:  $L_{Er,t}^b + B_{Er,t}^b > 0$ ,  $L_{C,t}^b + B_{C,t}^b < 0$ , and  $L_{\pi,t}^b + B_{\pi,t}^b \geq 0$ . Thus we have

$$(21) \quad C_t^b = C_t^b(r_{L,t}, E_t r_{C,t+1}, r_{C,t}; \pi_{t+1}^e), C_{L,t}^b > 0, C_{Er,t}^b > 0, C_{C,t}^b < 0, \\ C_{\pi,t}^b > 0 \text{ (if } \pi_{t+1}^e > 0), C_{\pi,t}^b < 0 \text{ (if } \pi_{t+1}^e < 0).$$

The rising (falling) loan rate, or a rise (fall) in the expected short-term rate that produces a rise (fall) of the bond rate, increases (decreases) call finance by the bank since the bank increases (reduces) an investment in the asset whose rate has become higher (lower). On the one hand, a rise (fall) in the call rate causes an increase (decrease) in the marginal cost of call finance; therefore, the fund raising in terms of call money declines (expands). On the other hand, when fluctuations of prices are

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equation (12). For reference, see Nagahara (2004, pp. 118-9).

expected, a rise in bond investments through the expansion of the spread leads to an increase in call money finance.

### 3.2 Firms

Assume that a representative firm invests in equipment by borrowing from banks and tries to maximize its profit from the investment. Then the firm's profit maximization problem is given as follows:

$$(22) \quad \begin{aligned} \text{Max.} \quad & \Pi_t^f = Q(I_t; \pi_{t+1}^e) - (1+r_{L,t})L_t^f - A_t^f(L_t^f), \\ \text{s.t.} \quad & I_t = L_t^f, \end{aligned}$$

where  $Q$  is a revenue function which is diminishing returns to an investment. Furthermore, suppose that the expected rate of inflation (deflation) is an exogenous parameter that has positive (negative) externalities for revenue, and that a firm's adjustment cost function,  $A_t^f(\cdot)$ , which stems from agency costs, is convex and twice continuously differentiable. Then the behavior of the firm is represented as follows:

$$(23) \quad I_t = L_t^f = L_t^f(r_{L,t}; \pi_{t+1}^e), \quad L_{L,t}^f < 0, \quad L_{\pi,t}^f > 0 \quad ^{14}.$$

A rise (fall) of the loan rate leads the firm to decrease (increase) the borrowing. When inflation (deflation) is anticipated, which implies an increase (decrease) of expected revenue in the future, the firm decides to expand (downsize) equipment; as a result, it increases (decreases) a borrowing.

### 3.3 Households

In our model the household deposits as much as it desires, given the fixed deposit rate  $r_{D,t}$ . The mathematical expression for this is as follows:

$$(24) \quad D_t^h = D_t^h(r_{D,t}).$$

However, the possibility of a change in  $r_{D,t}$  is not considered here; therefore, the behavior of the household is not explicitly dealt with later.

### 3.4 The Central Bank

Assume that the behavior of the central bank is formulated on the basis of both endogenous money supply theory, which Okina (1993) asserts, and the exogenous money supply. In other words, we suppose that commercial banks both supply credit in response to credit demand and create deposits, and that the central bank both

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14) For proof, refer to Nagahara (2003, pp.120-2).

accommodates the supply and demand of call money in the call money market, given the demand for reserves depending on the volume of deposits, and controls cash exogenously. Taking this into consideration, the central bank's behavior of endogenous money supply is represented by following the example of Yoshikawa (1992, p. 328)

$$(25) \quad \log C_t^{cb} = \eta (r_{C,t} - r_t^*),$$

where  $r_t^*$  is the level of the short-term rate that the central bank targets, and  $\eta \in [0, \infty]$  is a parameter that expresses the degree of its accommodativeness. A larger  $\eta$  means that the central bank behaves more accommodatively. Equation (25) can be rewritten as follows:

$$(26) \quad C_t^{cb} = C_t^{cb}(r_{C,t} | r_t^* = r_{C,t}), \quad C_{C,t}^{cb} \in [0, \infty]^{15}.$$

On the other hand, the central bank's exogenous money supply does not appear in the behavior equation (26). In contrast with the endogenous money supply, it is introduced as the rates of change in cash  $m_t$  in our model, which is described in the next section.

### 3.5 Financial Authorities

Suppose that financial authorities are faced with the following budget constraint at two-period

$$(27) \quad B_t^{fa} = G_t + (1 + R_t) B_{t-1}^{fa}.$$

Denote  $G_t$  as government spending at time  $t$ . Equation (27) shows in terms of nominal value that the government covers the current expenditure and the repayment of principal and interest of bonds issued at the previous period by issuing current bonds. Rewriting (27) as similar to (25) in the previous sub-section, we have

$$\log B_t^{fa} = \log G_t + R_t,$$

where  $\phi'$  is a parameter that represents the effects of a change in the long-term rate on an issue of current bonds, and a positive constant which depends upon the volume of bonds issued at time  $t-1$ ,  $B_{t-1}^{fa}$ , in equation (27).

Note that the long-term rate is expressed in the form of equation (8), and from (4) and (5) we have

$$(28) \quad \frac{\partial R_t}{\partial \pi_{t+1}^e} = \phi' \begin{cases} > 0 & \text{if } \pi_{t+1}^e > 0, \\ < 0 & \text{if } \pi_{t+1}^e < 0, \end{cases}$$

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15) While  $C_{C,t}^{cb} = 0$  shows that the call money supply curve of the central bank is vertical in  $(C, r_C)$  space,  $C_{C,t}^{cb} = \infty$  indicates it is horizontal there. Moore (1988, pp. x xi) calls the former "Verticalist" and the latter "Horizontalist".

and thus the behavior of financial authorities is specified as follows:

$$(29) \quad \begin{aligned} B_t^{fa} &= B_t^{fa}(E_t r_{C,t+1}, r_{C,t}; g_t, \pi_{t+1}^e), B_{Er,t}^{fa} > 0, B_{C,t}^{fa} > 0, B_{g,t}^{fa} > 0 \\ &B_{\pi,t}^{fa} > 0 (\text{if } \pi_{t+1}^e > 0), B_{\pi,t}^{fa} < 0 (\text{if } \pi_{t+1}^e < 0), \end{aligned}$$

where  $g_t \equiv \dot{G}_t / G_t$ . It is important to recognize that the condition  $B_{g,t}^{fa} > 0$  is directly obtained from (27), while the conditions  $B_{Er,t}^{fa} > 0$ ,  $B_{C,t}^{fa} > 0$ , and  $B_{\pi,t}^{fa} > 0$  are all derived from the indirect path of change in the long-term rate that stems from changes in the expected short-term rate, the current short-term rate, and the expected rate of inflation (deflation).

#### 4 EQUILIBRIUM IN THE GOODS MARKET AND RATIONALLY EXPECTED INFLATION

In the analysis so far, we have examined each agent's micro-foundations. In this section, we study how the expected inflation (deflation) is endogenously determined in the goods market from a macroscopic viewpoint. In other words, using the standard analysis of aggregate demand and supply<sup>16)</sup> allows us to deduce the way in which the expected prices react to exogenous shocks.

The aggregate demand in our model, in which household consumption and foreign trade are ignored, is given by

$$(30) \quad Y_t^d = F\left(\frac{M_t}{P_t}, G_t, I_t(\equiv L_t^f(r_{L,t}))\right).$$

We assume that function  $F(\cdot)$  can be rewritten in the form of a linear function as follows:

$$(31) \quad Y_t^d = \lambda \log(M_t/P_t) + \kappa \log G_t + \tau \log I_t + U_t,$$

where  $U_t$  is the error term with mean zero and variance  $\sigma_U^2$ . Differentiating (31) with respect to time and using  $\dot{Y}_t^d = Y_t^d - Y_{t-1}^d$ , we have the following inflation aggregate demand curve:

$$(32) \quad Y_t^d = Y_{t-1}^d + \lambda(m_t - \pi_t) + \kappa g_t + \tau i_t + u_t,$$

where  $m_t$ ,  $\pi_t$ ,  $g_t$  and  $i_t$  are the changes in rates of money supply, inflation, government spending, and equipment investment respectively; and  $\lambda$ ,  $\kappa$  and  $\tau$  are all positive structural parameters. Let  $u_t$  be a stochastic demand shock with  $u_t \sim N(0, \sigma_u^2)$  similar to  $U_t$  in (31).

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16) For example, see Romer (1996, ch.6) and Shone (2002, ch.11). It is well known that the model with stochastic disturbances is extended to the real business cycle approach.

On the other hand, assume the Lucas aggregate supply function as

$$(33) \quad \pi_t = \mu (Y_t^s - Y_F) + E_t \pi_t + v_t,$$

where  $Y_t^s$  is the current aggregate output,  $Y_F$  is the full employment output,  $E_t \pi_t$  is the expected rate of inflation determined at the beginning of time,  $t$ ,  $v_t$  is a stochastic supply shock with  $v_t \sim N(0, \sigma_v^2)$ , and  $\mu$  is a positive structural parameter.

From equations (32) and (33), the rate of inflation at equilibrium,  $\pi_t^*$ , is given by

$$(34) \quad \pi_t^* = \frac{1}{1 + \lambda \mu} \{ \mu [ Y_{t-1}^d - Y_F + \lambda m_t + \kappa g_t + \tau i_t + u_t ] + E_t \pi_t + v_t \},$$

and thus the inflation rate at equilibrium one-period forward,  $\pi_{t+1}^*$ , equals

$$(35) \quad \pi_{t+1}^* = \frac{1}{1 + \lambda \mu} \{ \mu [ Y_t^d - Y_F + \lambda m_{t+1} + \kappa g_{t+1} + \tau i_{t+1} + u_{t+1} ] + E_t \pi_{t+1} + v_{t+1} \}.$$

Now assuming agents' rational expectations, they expect the following with regard to the rate of inflation for the next period using the information set,  $\Omega_t$ , available at time  $t$ :

$$(36) \quad \begin{aligned} E_t \pi_{t+1} &= E_t (\pi_{t+1}^* | \Omega_t) \\ &= \frac{1}{1 + \lambda \mu} E_t \{ \mu [ Y_t^d - Y_F + \lambda m_{t+1} + \kappa g_{t+1} + \tau i_{t+1} + u_{t+1} ] + E_t \pi_{t+1} + v_{t+1} \} \\ &= E_t m_{t+1} + \frac{(Y_t^d - Y_F) + \kappa E_t g_{t+1} + \tau E_t i_{t+1}}{\lambda}. \end{aligned}$$

From equation (36), the effects of the current aggregate demand,  $Y_t^d$ , on an expected rate of inflation,  $\pi_{t+1}^e$ , become

$$(37) \quad \frac{\partial \pi_{t+1}^e}{\partial m_t} = \frac{\partial \pi_{t+1}^e}{\partial Y_t^d} \frac{\partial Y_t^d}{\partial m_t} = 1,$$

$$(38) \quad \frac{\partial \pi_{t+1}^e}{\partial g_t} = \frac{\partial \pi_{t+1}^e}{\partial Y_t^d} \frac{\partial Y_t^d}{\partial g_t} = \frac{\kappa}{\lambda} = \beta \geq 0,$$

$$(39) \quad \frac{\partial \pi_{t+1}^e}{\partial r_{L,t}} = \frac{\partial \pi_{t+1}^e}{\partial Y_t^d} \frac{\partial Y_t^d}{\partial r_{L,t}} = \frac{\tau}{\lambda} \cdot \frac{\partial i_t}{\partial r_{L,t}} = \gamma \leq 0.$$

Equation (39) has a negative sign because  $\partial i_t / \partial r_{L,t} < 0$  as shown in equation (23). While equations (37) and (38) reveal that expansionary (tight) fiscal or monetary policy leads to expectations of inflation (deflation) through an increase (decrease) in aggregate demand, equation (39) implies that the rising (falling) loan rate discourages (encourages) firms to invest in equipment, with the result that this diminishing (increasing) aggregate demand leads to expectations of deflation (inflation).

In summation, totally differentiating equation (36) with respect to  $\pi_{t+1}^e$ ,  $m_t$ ,  $g_t$ , and

$r_{L,t}$  yields

$$(40) \quad d\pi_{t+1}^e = dm_t + \beta dg_t + \gamma dr_{L,t}, \quad \beta \geq 0, \quad \gamma \leq 0.$$

## 5 GENERAL EQUILIBRIUM AND COMPARATIVE STATICS

### 5.1 Market General Equilibrium

In this section, we examine the general equilibrium system by making use of the results so far. Each market equilibrium is represented by equations (17), (18), (21), (23), (24), (26), and (29), respectively.

$$(41) \quad L_t^b(r_{L,t}, E_t r_{C,t+1}, r_{C,t}; \pi_{t+1}^e) = L_t^f(r_{L,t}; \pi_{t+1}^e), \quad (\text{loans market})$$

$$(42) \quad B_t^{fa}(E_t r_{C,t+1}, r_{C,t}; g_t, \pi_{t+1}^e) = B_t^b(r_{L,t}, E_t r_{C,t+1}, r_{C,t}; \pi_{t+1}^e), \quad (\text{bonds market})$$

$$(43) \quad C_t^{cb}(r_{C,t} | r_t^* = r_{C,t}) = C_t^b(r_{L,t}, E_t r_{C,t+1}, r_{C,t}; \pi_{t+1}^e), \quad (\text{call money market})$$

$$D_t^b = D_t^h(r_{D,t}), \quad (\text{deposits market})$$

$$Y_t^s = Y_t^d. \quad (\text{goods market})$$

As discussed in Section 2, we assume that the deposits market is always balanced. Thus, after eliminating the deposits market from the five markets, the remaining four markets become independent by applying Walras's Law. Then we focus on the following three: loans market, bonds market, and call money market.

With regard to the stability condition for market equilibrium, it is obvious that the loans and call money markets are Walras stable from the sign conditions of elasticity of each financial asset with respect to interest rates. On the other hand, although we cannot absolutely settle the stability condition of the bonds market because  $B_{Er,t}^b > 0$  and  $B_{Er,t}^{fa} > 0$ , we assume that the market is also stable and, therefore, that  $B_{Er,t}^b - B_{Er,t}^{fa} > 0$ . Moreover, it is proved by these assumptions that when  $\pi_{t+1}^e > 0$   $B_{\pi,t}^b - B_{\pi,t}^{fa} > 0$ ; and when  $\pi_{t+1}^e < 0$ ,  $B_{\pi,t}^b - B_{\pi,t}^{fa} < 0$  <sup>17)</sup>.

### 5.2 Comparative Statics

Totally differentiating equations (41)–(43) with respect to endogenous variables in the system  $r_{L,t}$ ,  $E_t r_{C,t+1}$ , and  $r_{C,t}$  and exogenous variables  $g_t$ ,  $m_t$ , and  $\pi_{t+1}^e$ , and

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17) Proof is as follows. By  $B_{Er,t}^b - B_{Er,t}^{fa} > 0$  that expresses the stability condition of bonds market,  $B_{\pi,t}^b - B_{\pi,t}^{fa} = \frac{\partial B_t^b}{\partial (E_t r_{C,t+1})} \cdot \frac{\partial (E_t r_{C,t+1})}{\partial R_t} \cdot \frac{\partial R_t}{\partial \pi_{t+1}^e} - \frac{\partial B_t^{fa}}{\partial (E_t r_{C,t+1})} \cdot \frac{\partial (E_t r_{C,t+1})}{\partial R_t} \cdot \frac{\partial R_t}{\partial \pi_{t+1}^e} = 2(B_{Er,t}^b - B_{Er,t}^{fa}) \times \frac{\partial R_t}{\partial \pi_{t+1}^e}$ . Therefore, the proof is established, by using (28) with respect to  $\partial R_t / \partial \pi_{t+1}^e$ .

inserting equation (40), we have the following simultaneous equations system<sup>18)</sup>:

$$\begin{aligned}
 (44) \quad & \begin{bmatrix} L_L^b - L_L^f - \gamma L_\pi^f & 0 & L_C^b \\ \gamma(B_\pi^b - B_\pi^{fa}) & B_{Er}^b - B_{Er}^{fa} & B_C^b - B_C^{fa} \\ C_L^b + \gamma C_\pi^b & C_{Er}^b & C_C^b - C_C^{cb} \end{bmatrix} \begin{bmatrix} dr_L \\ d(E_t r_{C,t+1}) \\ dr_C \end{bmatrix} \\
 & = \begin{bmatrix} \beta L_\pi^f \\ B_g^{fa} - \beta(B_\pi^b - B_\pi^{fa}) \\ -\beta C_\pi^b \end{bmatrix} dg + \begin{bmatrix} L_\pi^f \\ -(B_\pi^b - B_\pi^{fa}) \\ -C_\pi^b \end{bmatrix} dm.
 \end{aligned}$$

Let  $J$  be a  $(3 \times 3)$  matrix in the left hand side of equation (44); the Jacobian determinant  $|J|$  is defined as

$$\begin{aligned}
 (45) \quad |J| = & (L_L^b - L_L^f - \gamma L_\pi^f)(B_{Er}^b - B_{Er}^{fa})(C_C^b - C_C^{cb}) + L_C^b \gamma (B_\pi^b - B_\pi^{fa}) C_{Er}^b \\
 & - L_C^b (B_{Er}^b - B_{Er}^{fa})(C_L^b + \gamma C_\pi^b) - (L_L^b - L_L^f - \gamma L_\pi^f)(B_C^b - B_C^{fa}) C_{Er}^b.
 \end{aligned}$$

In our model, the behaviors of commercial banks and financial authorities change according to the sign of an expected fluctuation rate of prices, as shown in equation (18), (21), and (29). Hence, it is important to distinguish the difference of the sign positive or negative which is determined by the type of macroeconomic policy expansionary or tight. To clarify the results achieved later, assume that the call money supply curve of the central bank is horizontal; that is,  $C_C^{cb} \rightarrow \infty$ . However, since the call rate is given at the level that the central bank targets by this assumption, notice that it is the volume of call money, and not the call rate, that plays a role in adjusting supply and demand in the market.

### 5.2.1 Implementation of Fiscal Policy

The effects of fiscal policy on the short-and long-term rate are as follows:

$$\begin{aligned}
 (46) \quad \frac{dr_C}{dg} \Big|_{C_C^{cb} \rightarrow \infty} & = \frac{1}{|J|} \times \begin{vmatrix} L_L^b - L_L^f - \gamma L_\pi^f & 0 & \beta L_\pi^f \\ \gamma(B_\pi^b - B_\pi^{fa}) & B_{Er}^b - B_{Er}^{fa} & B_g^{fa} - \beta(B_\pi^b - B_\pi^{fa}) \\ C_L^b + \gamma C_\pi^b & C_{Er}^b & -\beta C_\pi^b \end{vmatrix} \\
 & = \frac{1}{|J|} \begin{bmatrix} (L_L^b - L_L^f - \gamma L_\pi^f)(B_{Er}^b - B_{Er}^{fa})(-\beta) C_\pi^b + \beta L_\pi^f \gamma (B_\pi^b - B_\pi^{fa}) C_{Er}^b \\ -\beta L_\pi^f (B_{Er}^b - B_{Er}^{fa})(C_L^b + \gamma C_\pi^b) \\ -(L_L^b - L_L^f - \gamma L_\pi^f) \{B_g^{fa} - \beta(B_\pi^b - B_\pi^{fa})\} C_{Er}^b \end{bmatrix}
 \end{aligned}$$

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18) For notational simplicity, drop in the later discussion a subscript that represents a point in time .

$$= 0^{19)}$$

Note that the long-term rate that is defined by (8) depends upon  $r_{c,t}$ ,  $E_t r_{c,t+1}$ , and  $\pi_t^e$  and we have

$$\begin{aligned}
 (47) \quad \frac{dR}{dg} \Big|_{C_C^{cb} \rightarrow \infty} &= \frac{\partial R}{\partial r_C} \cdot \frac{\partial r_C}{\partial g} + \frac{\partial R}{\partial (E_t r_{c,t+1})} \cdot \frac{\partial (E_t r_{c,t+1})}{\partial g} + \frac{\partial R}{\partial \pi^e} \cdot \frac{\partial \pi^e}{\partial g} \\
 &= \frac{1}{2|J|} \times \begin{bmatrix} L_L^b - L_L^f - \gamma L_\pi^f & \beta L_\pi^f & L_C^b \\ \gamma (B_\pi^b - B_\pi^{fa}) & B_g^{fa} - \beta (B_\pi^b - B_\pi^{fa}) & B_C^b - B_C^{fa} \\ C_L^b + \gamma C_\pi^b & -\beta C_\pi^b & C_C^b - C_C^{cb} \end{bmatrix} + \phi' \beta^{20)} \\
 &= \frac{1}{2|J|} \left[ \begin{aligned} & (L_L^b - L_L^f - \gamma L_\pi^f) \{B_g^{fa} - \beta (B_\pi^b - B_\pi^{fa})\} (C_C^b - C_C^{cb}) \\ & + L_C^b \gamma (B_\pi^b - B_\pi^{fa}) (-\beta) C_\pi^b \\ & + \beta L_\pi^f (B_C^b - B_C^{fa}) (C_L^b + \gamma C_\pi^b) \\ & - L_C^b \{B_g^{fa} - \beta (B_\pi^b - B_\pi^{fa})\} (C_L^b + \gamma C_\pi^b) \\ & - (L_L^b - L_L^f - \gamma L_\pi^f) (B_C^b - B_C^{fa}) (-\beta) C_\pi^b \\ & - \beta L_\pi^f \gamma (B_\pi^b - B_\pi^{fa}) (C_C^b + \gamma C_C^{cb}) \end{aligned} \right] + \phi' \beta \\
 &= \frac{-(L_L^b - L_L^f - \gamma L_\pi^f) B_g^{fa} + (L_L^b - L_L^f) \beta (B_\pi^b - B_\pi^{fa})}{-2(L_L^b - L_L^f - \gamma L_\pi^f) (B_{Er}^b - B_{Er}^{fa})} + \phi' \beta \\
 &= \frac{\underbrace{(L_L^b - L_L^f - \gamma L_\pi^f)}_{(+)} \underbrace{B_g^{fa}}_{(+)} - 4 \underbrace{\beta}_{(\geq 0)} \underbrace{\gamma}_{(\leq 0)} \underbrace{L_\pi^f}_{(+)} \underbrace{(B_{Er}^b - B_{Er}^{fa})}_{(+)} \underbrace{\theta}_{(\geq 0)} \underbrace{\pi^e}_{(+ \text{ or } -)}}{2 \underbrace{(L_L^b - L_L^f - \gamma L_\pi^f)}_{(+)} \underbrace{(B_{Er}^b - B_{Er}^{fa})}_{(+)}} \geq 0^{21)}.
 \end{aligned}$$

**Implications**

Firstly, equation (46) reveals that the short-term rate remains unchanged even when the fiscal policy is implemented. This is because a shock to the short rate is offset by the operations of the central bank.

Secondly, we cannot specify how the long-term rate changes in case of fiscal policy

19) Here, both the denominator and numerator are divided by  $C_C^{cb} \rightarrow \infty$ . This manipulation is not mentioned later as far as circumstances permit.

20) In the expansion from the first line to the second, we make use of equation (46) with regard to the first term at the right side, and of equations (28) and (38) with regard to the third term.

21) Here,  $B_\pi^b - B_\pi^{fa} = 2(B_{Er}^b - B_{Er}^{fa}) \phi' = 4(B_{Er}^b - B_{Er}^{fa}) \theta \pi^e$  mentioned in n.14 is used.



being implemented. This is because the sign of the numerator in (47) is not clearly known. We will discuss this further by distinguishing whether it is expansionary ( $\pi^e > 0$ ) or restrictive ( $\pi^e < 0$ ).

On the one hand,  $\pi^e > 0$  when an expansionary fiscal policy is conducted and thus, equation (47) is positive. In other words, the long-term rate rises due to the policy ( $g \uparrow \Rightarrow dR/dg > 0 \Rightarrow R \uparrow$ ). On the other hand, when the fiscal policy is restrictive, we cannot make a decision on the sign of (47), i.e., we still do not know the direction in which the long rate moves.

Let us focus on parameter  $\theta$ , which represents the extent of investors' response to a risk, and examine what happens to the above result. When  $\theta > 0$ , i.e., investors are surprised to a certain extent regarding the expected fluctuations of prices due to a change in policy, the second term in the numerator in (47) still remains; as a result, in the case of expansionary fiscal policy, the long-term rate rises; and in case of the tight one, we cannot specify the direction in which the rate fluctuates. This is the same result as above. In contrast, when  $\theta = 0$ , that is, they are not surprised by the fluctuations of prices,  $dR/dg > 0$  since the second term in the numerator in (47) is eliminated. In this case, therefore, the above results are modified, and the long-term rate falls in the event of a tight fiscal policy being implemented ( $g \downarrow \Rightarrow dR/dg > 0 \Rightarrow R \downarrow$ ).

## 5.2.2 Implementation of monetary policy

$$(48) \quad \left. \frac{dr_C}{dm} \right|_{C_C^{cb} \rightarrow \infty} = \frac{1}{|J|} \times \begin{vmatrix} L_L^b - L_L^f - \gamma L_\pi^f & 0 & L_\pi^f \\ \gamma(B_\pi^b - B_\pi^{fa}) & B_{Er}^b - B_{Er}^{fa} & -(B_\pi^b - B_\pi^{fa}) \\ C_L^b + \gamma C_\pi^b & C_{Er}^b & -C_\pi^b \end{vmatrix} \\ = 0,$$

$$(49) \quad \left. \frac{dR}{dm} \right|_{C_C^{cb} \rightarrow \infty} = \frac{\partial R}{\partial r_C} \cdot \frac{\partial r_C}{\partial m} + \frac{\partial R}{\partial (E_t r_{C,t+1})} \cdot \frac{\partial (E_t r_{C,t+1})}{\partial m} + \frac{\partial R}{\partial \pi^e} \cdot \frac{\partial \pi^e}{\partial m} \\ = \frac{1}{2|J|} \times \begin{vmatrix} L_L^b - L_L^f - \gamma L_\pi^f & L_\pi^f & L_C^b \\ \gamma(B_\pi^b - B_\pi^{fa}) & -(B_\pi^b - B_\pi^{fa}) & B_C^b - B_C^{fa} \\ C_L^b + \gamma C_\pi^b & -C_\pi^b & C_C^b - C_C^{cb} \end{vmatrix} + \phi' \\ = \frac{(L_L^b - L_L^f)(B_\pi^b - B_\pi^{fa})}{-2(L_L^b - L_L^f - \gamma L_\pi^f)(B_{Er}^b - B_{Er}^{fa})} + \phi'$$

$$= - \frac{2 \underset{(<0)}{\gamma} \underset{(+)}{L_{\pi}^f} \underset{(>0)}{\theta} \underset{(+)\text{or}(-)}{\pi^e}}{\underset{(+)}{(L_L^b - L_L^f - \gamma L_{\pi}^f)}} \stackrel{\text{22)}}{\approx} 0.$$

### Implications

First, similar to the case of fiscal policy, the short-term rate remains unchanged when monetary policy is conducted.

Second, the effects of the policy on the long-term rate depend on the sign of  $\pi^e$  in (49); thus, we have to determine whether  $\pi^e > 0$  or  $\pi^e < 0$ . When  $\pi^e > 0$ , which implies that expansionary policy has been put into practice, equation (49) is positive. In other words, the expansionary policy raises the long rate ( $m \uparrow \Rightarrow dR/dm > 0 \Rightarrow R \uparrow$ ). This result is similar to the expectations theory by Romer (1996, ch. 9), in which the term structure of interest rates is based on the Fisher identity<sup>23)</sup>. It is, however, necessary to note that the impacts of expected fluctuations of prices on the long rate have two routes in our model; one is the direct route induced by a change in the risk premium (the third term in the first line of the right hand side of (49),  $[\partial R/\partial \pi^e] \times [\partial \pi^e/\partial m]$ ), and the other is the indirect path that a change in the premium causes commercial banks to adjust their behavior and that the adjustment leads to fluctuations of the expected short-term rate that result in a change of the long-term rate (the second term there,  $[\partial R/\partial (E_t r_{C,t+1})] \times [\partial (E_t r_{C,t+1})/\partial m]$ ). It can be concluded that our result is because the former route has a stronger effect on the long-term rate than the latter. On the other hand, when a tight monetary policy is conducted and  $\pi^e < 0$ , equation (49) is negative; that is, the long-term rate rises in this case as well ( $m \downarrow \Rightarrow dR/dm > 0 \Rightarrow R \uparrow$ ). This is opposite to the result by Romer (1996).

Third, comparative statics with respect to a parameter  $\theta$  that represents an investors' response to a risk is as follows: When the monetary policy surprises the investors, the long-term rate rises, while it remains unchanged when they are not surprised at the policy. The following can be stated in relation to the asymmetry of

22) These mathematical manipulations utilize n. 17 and n. 18.

23) According to Romer (1996), an exogenous monetary shock moves the short-term rate and the long-term rate in the opposite direction. This is because a liquidity effect that influences the short rate and an expected inflation effect that influences the long rate conflict with each other. For example, when expansionary monetary policy is put into operation, the short rate gets lower due to supply and demand of liquidity being relaxed, while the long rate gets higher due to the expectations of inflation.

monetary policy effects: Although restrictive monetary policy is very effective against an overheated economy, monetary easing does not work well in a downturn<sup>24</sup>). Even though the central bank conducts expansionary policy to lower the long-term rate in a recession, it gets higher when the policy surprises investors, so that the monetary easing effect becomes weak. Taking into consideration that, in general, the implementation of policy brings economic agents considerable surprise, the central bank has to pay as close attention to putting expansionary monetary policy into operation lest the policy causes them surprise; for example, it needs to fulfill accountability regarding the conducted policy.

## 6 CONCLUDING REMARKS

In this paper we have examined the effects of macroeconomic policy on the long- and short-term rates in the system in terms of the market equilibrium analysis. We have assumed that the nominal yield on two-period bonds equals an average of the current short-term rate and the subsequent short-term rate plus an increasing risk premium with respect to expected fluctuations of prices. We also have assumed that the expectations of inflation (deflation) are endogenous and are rationally determined in the good market.

In summary, we have achieved the following conclusions.

First, as regards the effects of monetary policy on the long-term rate, our results differ from those of the expectations theory of the term structure by Romer (1996). In other words, while the model in Romer (1996) states that tight monetary policy lowers the long rates whereas expansionary policy raises them, our model concludes that restraint monetary policy raises the rate and that easing monetary policy also tends to arise it. This is because the effects, which a change in expectations exerts on the long rate, are supposed to be different between both models. In detail, Romer (1996) considers that the change in expectations extends only to the expected short-term rate, which is  $E_t r_{C,t+1}$  on the basis of equation (8), and with the result that long-term rates move. On the other hand, we suppose dual effects of the change on the long rate: It spreads not only the expected short-term rate, but also the risk premium

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24) "By contrast with its effectiveness in periods of boom ..., monetary policy has often been described as ineffective in a recession." (Stiglitz and Greenwald, 2003: p.130)

$\phi_t = \theta(\pi_{t+1}^e)^2$  in (8).

Second, the impacts of fiscal policy on the long-term rate are that expansionary fiscal policy causes a rise in the long-term rate; however, the direction in which the rate moves is unknown in case of tight policy being conducted. However assuming that  $\theta = 0$ , that is, investors are not surprised, it is shown that the rate gets lower by tight policy.

Third, monetary or fiscal policy exercises no influence on the short-term rate in the system, as long as  $C_c^{cb} \rightarrow \infty$ , i.e., the central bank completely accommodates any disturbance in the call money market<sup>25)</sup>.

Finally, the degree of investors' response to a risk,  $\theta$ , has crucial effects on the long-term rate. For example, as shown in the previous section, expansionary monetary policy becomes effective in the sense that the long rate does not rise<sup>26)</sup>, if and only if  $\theta = 0$ . This has two implications. One is practical and that the BoJ must fulfill accountability toward its policy lest it causes negative surprise among agents. This is still significantly important, as stated earlier, because the focus is on the trend of long rates through various factors in the current and future Japanese economy. The other is theoretical, namely, the results from the simulation above greatly depend on  $\theta$  and, therefore, on the formulation of the risk premium function defined in equation (4). In this respect, preceding studies, such as funds flow models mentioned in Shiller (1990, p. 668), must be instructive for our future works.

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25) For generality, it is essential to remove this assumption. This will be a topic for our future study.

26) The rise in long-term rates is a factor of decreasing aggregate demand; however, this is not explicitly shown in this model.

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