On Debt Cycles

Ryuzo KUROKI

Abstract : This paper will explore the reciprocal interaction between the real and the financial sectors and examine the role of credit and the effect of borrowing and debt burden on the level of output. In our model, the representative firm takes into account the level of the interest rate, economic activities and its debt burden, when investing. Banks play a role in transmitting financial facilities to the firm and the central bank in regulating the level of the interest rate.

1. Introduction — Money and Credit

One of the most important subjects of economic study is of course the monetary aspect : money and credit are two of the most indispensable factors of any effectively functioning economy. With a shortage of money and credit, our economy has sometimes suffered serious difficulties accompanied by the underemployment of resources, for instance in the unemployment of labor.

In this paper, we will examine the dynamic properties of the macroscopic economy introducing a simple one sector model which embodies the financial structure. Referring to Minsky's idea, we will develop the macro model wherein the representative firm adjusts its investment to changes in the financial market. One of the main points of this study is to provide the role of monetary authority, that is, of the central bank, which manipulates the level of the interest rate to control economic fluctuations.

2. Business Cycles Derived from IS-LM Model

With respect to the dynamical stability of the macroscopic economy, the textbook proposition rests on the assumption that the aggregate demand function must be downward sloping in terms of the level of national production (Y) and the interest rate (r). But it is well known that even in the introductory framework we can easily show an indication of cyclical instability or even a saddle point property. Let us take a look at the dynamical system derived from the follwing simple model :

$$dY/dt = I(Y, r) - S(Y), \quad dr/dt = L(Y, r) - M \tag{1}$$

Here, the investment I is an ordinary decreasing function of the interest rate r, but, on the other hand, it is also supposed to be an increasing function of the output level Y. S is a normal saving function, which reacts to the level of Y positively, and LM curve also has an ordinary shape. The dynamic property of this model could be formulated as follows :

$$\partial I/\partial Y > 0,\ \partial I/\partial r < 0,\ \partial S/\partial Y > 0,\ \partial L/\partial Y > 0,\ \partial L/\partial r < 0,\ M = M^{*}(\text{constant}) \tag{2}$$

With respect to the trace of Jacobian matrix of this system, we have the following :

$$trace\ J = (\partial I/\partial Y) - (\partial S/\partial Y) + (\partial L/\partial r) \tag{3}$$

Provided that the determinant $J > 0$, if the trace $J < 0$, then the system is stable. But if $tr\ J > 0$, then the fixed point of our system will become unstable. We could say that the dynamic property of the system depends solely on how the firms form their volume of investments, paying attention to the level of output ($\partial I/\partial Y$). If we have ($\partial I/\partial Y$) which is sufficiently large in the middle region of the output level, but is small enough in the low and in the high level of output, then we could even have a cyclically divergent movement between the level of output and the interest rate. Moreover, if the sign of the determinant J becomes negative due to the sufficiently large $\partial I/\partial Y$, then we will have a saddle point or have a possibility of the existence of multiple equilibria.[1]

3. The Debt Accumulation Generated from the Balance Sheet

3-1. The dual dynamical system

Let us consider the financial structure of the representative firm which finances investments and production costs. The firm spends its sales to buy materials, to hire workers and to invest. When its funds short, the firm asks banks for the borrowing and sometimes issues bonds or equities to collect money. Here, we assume first that the firm finances investments by means of retained earnings and the borrowing (B) from the banks. The firm has a linear production function with respect to the labor input N. The firm gets the gross profit as the difference of the amount of sales and the wage bill. It has to pay interests and principal due when it owes the debt burden to the banks. If the firm has the bank balance, then it could receive interest payments. So that, taking into account the financial constraint, we have the dual system of the change of output and the debt accumulation.[2] In the following, w is the wage rate (constant), l is the labor-input coefficient (constant), r (>0) is the interest rate, and e is the rate of principal due (constant ≥ 0).

$$\begin{aligned} \text{Borrowing from banks}: B &= I + wlY - Y + rD + eD \quad (\text{if } D > 0) \\ B^{*} &= I + wlY - Y + rD \quad (\text{if } D \leqq 0) \qquad (4) \end{aligned}$$

Letting L represent wage costs per one unit of production ($L = wl$),

$$\begin{aligned} \text{Debt accumulation}: dD / dt &= B - eD \quad (\text{if } D > 0) \\ &= B^{*} \quad (\text{if } D \leqq 0) \\ &= I - Y\,(1 - L) + rD \qquad (5) \end{aligned}$$

Assume that all the wages are spent on consumption ($C = wlY = LY$), then we have

$$\text{Change of the output}: dY / dt = I + C - Y = I - Y\,(1 - L) \qquad (6)$$

1) See, for example, Zhang,W. B. [3].

2) See Jarsulic, M. [1] and for more detailed discussions, see Semmler, W. [2].

3-2. The Investment function

The main driving factor of our model rests on how the firm forms its investment. The discounted value V of the flow of prospective yield $Q(t)$ brought from the investment, will be expressed by

$$V = \sum_{t=1}^{n} Q(t) / (1+r+\sigma(D))^t \tag{7}$$

σ in the denominator of the right side represents the risk-premium which consists of the difference of the rates of return between risky assets (=investments) and no risk assets (=government bonds or term deposits). Since the more the debt burden grows, the higher the probability of bankruptcy will become (=default risk), we assume that σ is an increasing function with respect to the debt-burden :

$$\sigma = \sigma(D), \ \partial\sigma / \partial D > 0 \tag{8}$$

Let Q represent the average prospective yield and substitute the following for (7);

$$V = \sum_{t=1}^{n} Q(t) / (1+r+\sigma(D))^t \cong Q / (r+\sigma), \quad n < \infty \tag{9}$$

Q is an increasing function with respect to the investment I and the marginal efficiency of investment $\rho = \partial Q / \partial I$ (>0) decreases as I increases ($\partial\rho / \partial I < 0$). If we suppose that the prospective yield Q is an increasing function with respect to the level of output Y, the marginal efficiency of investment ρ increases as the output level becomes large, and the firm will maximize the discounted value of the net cash-flow expected from the investment, π, then we have

$$\max.\ \pi = \max.\ (V-I) = \max.\ \{Q(Y, I) / (r+\sigma)\} - I$$

$$\text{the first order condition} : \partial Q / \partial I \ (=\rho) = r+\sigma \tag{10}$$

Differentiate (10) and we have

$$(\partial\rho / \partial I)\, dI + (\partial\rho / \partial Y)\, dY = dr + (\partial\sigma / \partial D)\, dD \tag{11}$$

Consequently the investment function is obtained as a function of the interest rate r, the debt burden D and the output level Y, as follows:

$$I = I\ (r,\ D,\ Y),\ \partial I / \partial r < 0,\ \ \partial I / \partial D < 0,\ \ \partial I / \partial Y > 0 \tag{12}$$

4. Monetary Policy and Business Cycles

4-1. Monetary intervention

The interest rate r is assumed to be set by the monetary authority (the central bank) partly exogenously and partly endogenously. That is,

$$r = a + bY$$

$a > 0$ is an exogenous parameter set by the central bank, and

$b > 0$ is constant, showing that the central bank's policy responds to changes in the level of activity. (13)

The parameter a in (13) may be considered to represent a strong intention of the monetary authority and the parameter b represents an adjustment which raises the interest rate or lowers it according to the level of output. So, the interest rate could be regarded as a kind of strategic manipulating variable with which the monetary authority tries to intervene in the financial market to control business fluctuations.

4-2. Shift of an equilibrium point and its stability

To observe is that the firm does not owe any substantial debt to the banks but has net assets with them when its debt burden is negative. From the formulation above, we have the fixed point presented by $Y = Y^* = I/(1-L) = I/(1-c)$, and $D = D^* = 0$, the former of which will reconfirm the theory of Keynesian multiplier.

Now let us study the dynamic characteristic of our system. The system shown above would be rewritten as follows;

$$dY/dt = I + C - Y = I - Y(1-L) = f_1\ (Y,\ D : a) \tag{14}$$

$$dD/dt = I - Y(1-L) + rD = f_2\ (Y,\ D : a) \tag{15}$$

Our system could be approximately substituted by the following first order Taylor expansion :

$$dY / dt = f_1 \ (Y,\ D : a) = f_{11} \ (Y - Y^*) + f_{12} \ (D - D^*) \tag{16}$$

$$dD / dt = f_2 \ (Y,\ D : a) = f_{21} \ (Y - Y^*) + f_{22} \ (D - D^*) \tag{17}$$

1)～3) cases shown below are related to the components of the Jacobian matrix.

1) The first case

If the investment depends solely on the level of interest rate ;

$$I = I\,(r),\ \ \partial I / \partial r < 0 \tag{18}$$

Then, we have

$$\begin{aligned}
&f_{11} = (\partial I / \partial r)(\partial r / \partial Y) - (1 - L) = (\partial I / \partial r)\ b - (1 - L) < 0 \\
&f_{12} = 0 \\
&f_{21} = (\partial I / \partial r)(\partial r / \partial Y) - (1 - L) + D^*(\partial r / \partial Y) = (\partial I / \partial r)\, b - (1 - L) < 0,\ \because D^* = 0 \\
&f_{22} = r = a + bY > 0
\end{aligned} \tag{19}$$

1)-(i) Stability analysis

The Characteristic equation of the system can be shown as follows :

$$|\lambda I - J| = (\lambda - \alpha)(\lambda - \beta) = \lambda^2 - (\alpha + \beta)\ \lambda + \alpha\beta \tag{20}$$

The sign and the magnitude of the roots λ determine the nature of the dynamic system. Here, let us examine the property of the fixed point :

$$\begin{aligned}
\alpha + \beta &= tr\ J = f_{11} + f_{22} \\
&= (\partial I / \partial r)(\partial r / \partial Y) - (1 - L) + r = (\partial I / \partial r)\ \ b - (1 - L) + a + bY \\
\alpha\beta &= det\, J = f_{11} f_{22} - f_{12} f_{21} < 0.
\end{aligned} \tag{21}$$

Thus, this system has one positive real root and one negative real root, which means that the fixed point becomes saddle.

1)-(ii) Effect of the monetary intervention — comparative statics

Differentiate (14) and (15) with respect to Y, D, and the parameter a in equilibrium. Then, we get

$$\{(\partial I/\partial r)\ b-(1-L)\}\ dY+0dD+(\partial I/\partial r)\ da=0$$
$$\{(\partial I/\partial r)\ b-(1-L)\}\ dY+(a+bY)\ dD+\{(\partial I/\partial r)+D\}\ da=0 \tag{22}$$

Consequently, taking care that in equilibrium, $D^*=0$, we have

$$dY/da=-(\partial I/\partial r)(a+bY)/\det J<0,\quad dD/da=0/\det J=0 \tag{23}$$

If the investment depends solely on the interest rate, the easy money policy of the monetary authority $(da<0)$ will make the production level increase, that is, $dY>0$ (which seems to be a normal result).

2) The second case

Let us take Kalecki's and Minsky's ideas and let the investment function depend not only on the interest rate but also on the level of production Y and on the debt burden D:

$$I=I\ (r,\ Y,\ D),\ \partial I/\partial r<0,\ \partial I/\partial Y>0,\ \partial I/\partial D<0\ \text{(see (12))}$$

Then, we get the following;

$$\begin{aligned}
f_{11} &= (\partial I/\partial r)(\partial r/\partial Y)+(\partial I/\partial Y)-(1-L)\\
&= (\partial I/\partial r)\ b+(\partial I/\partial Y)-(1-L)\\
f_{12} &= \partial I/\partial D<0\\
f_{21} &= (\partial I/\partial r)(\partial r/\partial Y)+(\partial I/\partial Y)-(1-L)+D^*(\partial r/\partial Y)\\
&=(\partial I/\partial r)\ b+(\partial I/\partial Y)-(1-L)\\
f_{22} &= (\partial I/\partial D)+r=(\partial I/\partial D)+a+bY
\end{aligned} \tag{24}$$

In this case, we may have some complicated results. That is,

$$\begin{aligned}
tr\,J &= (\partial I/\partial r)\ b+(\partial I/\partial Y)-(1-L)+(\partial I/\partial D)+a+bY\ (?)\\
\det J &= \{(\partial I/\partial r)\ b+(\partial I/\partial Y)-(1-L)\}\{(\partial I/\partial D)+a+bY\}\\
&\quad -(\partial I/\partial D)\{(\partial I/\partial r)\ b+(\partial I/\partial Y)-(1-L)\}\\
&= \{(\partial I/\partial r)\ b+(\partial I/\partial Y)-(1-L)\}(a+bY)\ (?)
\end{aligned} \tag{25}$$

The firm will become risk averse as its debt burden to the banks drastically accumulates and hence will refrain from investments in proportion to the growth of the debt. Therefore, we will study the case in which investments will decrease as the debt becomes large.

Note that as the dynamical properties of this model, three cases may arise. The first case: the element f_{11} is smaller than zero. The second: f_{11} becomes larger than zero. And in the third case, f_{11} becomes equal to zero. If $f_{11}<0$, then, $det J$ becomes negative and the fixed point (Y^*, D^*) will exhibit the saddle.

In the following, we examine the case of f_{11} (implying f_{21} too) being larger than zero, and consequently, of $det J>0$, and we will concentrate on the case in which cyclical movements of the variables are observed.

3) Cycles

Assume that the degree of the decrease in investment due to the increase in the debt burden (that is, the absolute value of $(\partial I/\partial D)$) is sufficiently large, and f_{22} is negative. Assume also that the level of output has a positive effect on the investment decision, which is large enough to let f_{11} be positive. Finally, assume that the monetary authority can manipulate the parameter a as the policy variable and that a^* exists which generates a pair of pure imaginary roots.

3)-(i) Effect of the monetary intervention — comparative statics

Take care that $I=I(r, Y, D)$, $\partial I/\partial r<0$, $\partial I/\partial Y>0$, $\partial I/\partial D<0$, and differentiate (14) and (15) with respect to Y, D, and the parameter a in equilibrium. Then, we get

$$\{(\partial I/\partial r)\, b+(\partial I/\partial Y)-(1-L)\}\, dY+(\partial I/\partial D)\, dD+(\partial I/\partial r)\, da=0,$$

$$\{(\partial I/\partial r)\, b+(\partial I/\partial Y)-(1-L)\}\, dY+\{(\partial I/\partial D)+a+bY\}\, dD + \{(\partial I/\partial r)+D^*\}\, da=0 \tag{26}$$

Consequently, we have

$$dY/da=-(\partial I/\partial r)(a+bY)/det J>0, \quad dD/da=0 \tag{27}$$

If the investment depends not only on the level of the interest rate, but also on the output level and on the debt burden, then, in this case, the easy money policy of the monetary authority could cause the output level of the fixed point to decrease.

3)-(ii) On dynamics

When we have easy money policy, that is, when the interest rate is cut, tr J becomes negative as a decreases, and we have a cyclical stability. As the authority changes their policy towards tightening and a grows, the system gradually changes its dynamic features, and eventually becomes cyclically instable. When a^* which generates a pair of pure imaginary roots is adopted, if $\partial \ tr \ J / \partial a^* \neq 0$, there will exist the limit cycle (Hopf bifurcation).

5. Concluding remarks

In the credit-driven capitalist economy, if the monetary authority can manipulate the interest rate, it will have some influence upon the characteristics of the equilibrium.

In the case of the system where we have a pro-cyclical (self-augmenting) change with respect to the output level, if the monetary authority adopts an easy money policy, reducing the parameter a, the level of the equilibrium point will decrease and its dynamical feature will become stable. We could probably infer from this analytical result that an easy monetary policy will accelerate investment, which causes the debt burden to accumulate, and a rise of the debt level will bring a shrinking of investment and therefore will finally lead the output level to decrease.

As policy tightens and the parameter a enlarges, the system will become destabilized in a cyclical manner.

References

[1] Jarsulic, M., “Debt and Macro Stability,” *Eastern Economic Journal*, vol.XV, no.3, 1990.

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[3] Zhan, W., *Synergetic Economics : Time and Change in Nonlinear Economics*, 1991, Springer-Verlag.