The Hedonic MDP Procedures for Quality Attributes: Optimality and Incentives

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ABSTRACT: This paper presents *hedonic MDP Procedures* for adjusting quality attributes by applying the methods employed to formulate the MDP and the Generalized MDP Procedures for public goods. In doing so, I adopt an analytical framework of the "new consumer theory" taken initiative by Gorman and Lancaster, and made rigorous by Drèze and Hagen. Also used is Sen's capability theory to fully appraise through individuals' functionings the value of each attribute compounded in the goods. It is verified that our procedures can satisfy some normative conditions including the "Samuelsonian Hedonic Conditions" for goods' attributes which can be regarded as public goods.

Key Words: Generalized ζ MDP Procedures, Gorman-Lancasterian characteristics, Hedonic MDP Procedures, Samuelsonian Hedonic Conditions, Sen's capability and functionings.

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1. INTRODUCTION

1.1. The issue of the present paper is to design hedonic planning procedures for adjusting the quality of goods which can be considered as a combination of quality characteristics. In an attempt to confront and clarify the problem, I adopt an analytical framework of the New Consumer Theory and the Capability Theory. It is verified that the necessary conditions for Pareto optimal product quality in terms of Gorman-Lancasterian attributes, with which our procedures are constructed.

The *New Consumer Theory* was initiated by Gorman(1956/1980)[1]. It was followed by Lancaster(1966) and (1971) and was rigorously analyzed by Drèze and Hagen(1978). Fundamental theorems of welfare economics state that any competitive equilibrium is Pareto optimal that holds for each good, but not for each attribute or characteristic with which goods are composed. This observation was proven by Hagen(1975) in his new consumer theoretical framework. Drèze and Hagen(1978) subsequently developed a model based on the New Consumer Theory, drove necessary conditions (or conditions for Pareto stationary points) for optimal product quality in a general equilibrium system, and verified a simultaneous establishment of quantitative and qualitative efficiency. They also proved that producers who maximize their profits have an incentive to select the most desirable combination of attributes which maximizes consumer's utility. That is, they demonstrated that the production of goods having Pareto optimal product quality is compatible with the profit maximizing behavior of producers. They analyzed two equilibrium concepts: monopolistic Nash equilibrium and competitive profits equilibrium. Furthermore, they drove a Slutsky equation for quality changes.

The *objective* facets of the goods choice problem also involves *subjective* considerations, since consuming behaviors are very personal activities. Hence, I also adopt *Amartya Sen's Capability Approach*, which is used to value individual well-being via personally optimal selection of goods. Owing to the available characteristics embodied in goods, people can live a life consisting of "beings" and "doings". Our aim is to show that any consumer can maximize his or her "happiness function" by optimally selecting and consuming goods combinated by numerous attributes. Thus, composing an optimal goods selection is to be one of the maximal elements in his/her *capability set* in terms of Sen, to be explained below.

1.2. Samuelson's optimality conditions for public goods, first propounded in 1954, has acquired universal familiarity. These conditions, however, implicitly assumed a *fixed quality* of public goods and focused upon the quantitative efficiency. Drèze and Hagen(1978) generalized the optimality conditions which hold for both goods and characteristics, and I extend furthermore their results in our characteristics/functionings framework.

My concern in this paper is to construct planning procedures, i. e., an extention of the original MDP Process, which is reachable to even qualitative optima. In an attempt to solve the issue, the present paper verifies the "Samuelsonian hedonic conditions(SHC)", with which the procedures are designed. This paper is a follow up on the literature on the use of planning procedures as mechanisms for aggregating the decentralized information needed for guiding public decisions, e. g., Malinvaud(1970–71), (1971) and (1972), Drèze and de la Vallée Poussin(1971), Fujigaki and Sato(1981) and (1982). Recognizing the difficulties related to the possibility of manipulation of information by individual agents, this literature has shown that this problem could be dealt with by such planning procedures that require a continuous revelation of information, provided that agents adopts a myopic behavior.

I combine good "characteristics" of the above strains of research to establish efficiency conditions for the products composed of characteristics. As a result, I derive the necessary and sufficient conditions for consumers to maximize their "happiness function" by choosing Pareto optimal good quality represented by characteristics embodied in goods. Also deduced are the necessary conditions for producers to maximize their profits by providing Pareto efficient good quality attributes.

This paper proceeds as follows. Section 2 presents the model which includes Gorman/Lancasterian characteristics and Sen's capabilities, and also discusses the valuation of the personal well-being. The necessary and sufficient conditions for an individually optimal selection of goods as a composition of attributes embodied in products are also derived in Section 2. The Samuelsonian hedonic conditions are verified and the procedures based on SHC are presented in Section 3. Section 4 confirms that the normative conditions are fulfilled by the processes: feasibility, monotonicity, stability, neutrality, and incentive properties pertaining to minimax and Nash equilibrium strategies. Existence and stability of the trajectories are proved for these planning algorithms. Three conditions are introduced: e. g., aggregate correct revelation, transfer anonimity and transfer neutrality to axiomatize the processes. Then I characterize the nonlinearized and generalized ζMDP Procedures, and examine their properties including local strategy proofness. Proofs of the main theorems are presented in Section 5. Section 6 concludes this paper with some final remarks.

2. THE CHARACTERISTICS-CAPABILITY MODEL

2.1. *Goods and Characteristics*

Let there be N consumers indexed by $i \in \boldsymbol{N} = \{1,\ldots, N\}$: the set of individuals. I regard goods as composed of C characteristics indexed by $c \in \boldsymbol{C} = \{1,\ldots, C\}$: the set of attributes. Two terms, *attributes* and *characteristics*, are used interchangeably throughout the paper. Denote q_{jc} as an amount of attribute c embodied in one unit of good x_j. Let $\boldsymbol{J} = \{1,\ldots, J\}$ be the set of goods. As in Drèze and Hagen(1978), every good is assumed to have at least one characteristic indexed by j', hence other attributes are measured per unit of characteristic j', which may differ among goods. We impose $q_{jj'} = 1$ for size normalization. This subsection is based on their analysis.

Denote q as the $\boldsymbol{J}\times\boldsymbol{C}$ (variable) "technology matrix" with typical element, q_{jc}. Let x_{ij} be a person i's consumption of good j, and let $x_i = (x_{i1},\ldots, x_{iJ})$ be his/her consumption vector. Each consumer has a consumption set $\boldsymbol{Z}_i$ in the space of attributes and the numéraire. The initial resources of individual i are defined by a nonnegative vector, $\omega_i = (\omega_{i0}, \omega_{i1},\ldots, \omega_{iC})$. The sale or purchase of the commodities by individual i is x_{i0}.

Let z_{i0} be an individual i's *numéraire characteristic* that he/she possesses, by which any attribute is utilized through his/her functionings. The consumption of individual i in the space of attributes and the numéraire is

$$z_i = (z_{i1},\ldots, z_{iC}) = \omega_i + (x_{i0}, x_i' q)' \tag{1}$$

where amounts of each characteristic embodied in goods consumed by individual i is given by

$$z_{ic} = \sum_{j=1}^{J} x_{ij} q_{jc},\ \forall c \in \boldsymbol{C}. \tag{2}$$

Equation (2) may be interpreted as a *characteristics availability function*, that converts commodities into attributes. I consider q_{jc}, $\forall j \in \boldsymbol{J}$, $\forall \boldsymbol{c} \in \boldsymbol{C}$, as parameters that are objective and common to all consumers, i. e., they have a public-good property.

However, only producers can vary q_{jc} by their production technologies, but consumers cannot. They can change their consumption of attributes by varying their consumption vector, x_i.

Let there be J producers, indexed by $j \in \boldsymbol{J} = \{1,\ldots, J\}$: the set of producers. Each firm j produces good j by using an input x_{j0}, and has a production set $\boldsymbol{Y}_j$ in the space of a product and its attributes. Denote $y_j = (x_{j0}, x_j, q_{j1},\ldots, q_{jC})$ be an input-output vector.

The production function is represented by

$$(3) \qquad \upsilon_j(x_{j0}, x_j, q_{j1},\ldots, q_{jC}) = 0$$

with $x_j \geqq 0$ and $q_{jc} \geqq 0$, $\forall j \in \boldsymbol{J}$, $\forall c \in \boldsymbol{C}$. The producers sell their product x_j at the price p_j, and a price of x_{j0} is normalized to unity.

Here I need to make an assumption.

Assumption 1: For any $j \in \boldsymbol{J}$, υ_j is convex and twice continuously differentiable on the closed convex production set $\boldsymbol{Y}_j$, with $\partial x_{j0}/\partial q_{jc} > 0$, $\forall j \in \boldsymbol{J}$, $\forall c \in \boldsymbol{C}$, $c \neq j'$. Moreover, $x_j > 0$ implies $x_{j0} > 0$, and $\forall \Lambda \in \boldsymbol{R}_+$, $\{y_j \mid \upsilon_j(y_j) \leqq 0, x_{j0} \leqq \Lambda\}$ is compact.

2.2. *Beings and Functionings*

Diverse beings are attained by utilizing functionings with which consumers use commodities available to them. A person does not necessarily buy nor possess all of the goods that he or she uses. It is enough for an individual to have only access to the commodities which are necessary. Hence, refering to x_{ij}, this does not imply that the vector of goods, x_i, is possessed by person i, but rather just available to i. A person's state of being is understood as a vector of functionings. The set of feasible vectors of functionings for any person is his/her *capability set*, i. e., opportunities to achieve whatever beings he or she can choose. Before rushing into the results, let me introduce some basic concepts different from Sen(1985). His theory seems to assume a one-to-one correspondence between a being and a functioning. It is more natural, however, to think that an individual i's *being* is generated by a simultaneous utilization of his/her functionings, f_{ik}, $k = 1,\ldots, K_i$, which may be represented by

$$(4) \qquad b_i = (f_{i1}(z_i),\ldots, f_{iK_i}(z_i)),\ \forall k_i \in \boldsymbol{K}_i.$$

where $\boldsymbol{K}_i = \{1,\ldots, K_i\}$ is the set of person i's functionings. The number of functionings, K_i, varies among individuals. Let me make an assumption.

Assumption 2: For any $i \in \boldsymbol{N}$, f_{ik} is strictly quasi-concave and twice continuously differentiable on the closed convex consumption set $\boldsymbol{Z}_i$.

Remark 1: Suppose that change in the numéraire attribute, z_{i0}, can vary person i's functionings. The sign of $\partial f_{ik}/\partial z_{ic}$ depends upon what characteristic c is, i. e., it can take a sign $\{+, 0, -\}$ according to c which is good (irrelevant, bad, respectively) for person i's functionings.

Let $\boldsymbol{X}_i$ be the set of vector of goods available to an individual i. Given $\boldsymbol{X}_i$, I can represent the set of feasible functionings vector, or the *capability set* of person i as:

$$\text{(5)} \qquad \boldsymbol{B}_i(\boldsymbol{X}_i) = \{b_i \mid b_i = (f_{i1}(z_1), \dots, f_{iK_i}(z_i)),\ \textit{for some}\ k_i\ \textit{in}\ \boldsymbol{K}_i\ \textit{and for some}\ x_i\ \textit{in}\ \boldsymbol{X}_i\}.$$

2.3. *Happiness Function and Valuing Individual Well-Being*

According to Sen(1985), person i's "*Happiness Function*" is assumed to depend upon his/her beings. Thus, one has

$$\text{(6)} \qquad H_i = H_i(b_i).$$

In order to deepen his analysis to obtain the desired results, I need the differentiability assumption. It is natural to consider that change in any functioning of a person can vary his/her happiness, so that the following assumption is imposed.

Assumption 3: *For any* $i \in \boldsymbol{N}$, H_i *is strictly quasi-concave and twice continuously differentiable. There is at least one* $k_i \in \boldsymbol{K}_i$ *such that* $(\partial H_i/\partial f_{ik})(\partial f_{ik}/\partial z_{i0}) \neq 0$.

Here we introduce a new concept. Denote an *individual hedonic price* of attribute c as:

$$\text{(7)} \qquad \pi_{ic} = \frac{\sum_k (\partial H_i/\partial f_{ik})(\partial f_{ik}/\partial z_{ic})}{\sum_k (\partial H_i/\partial f_{ik})(\partial f_{ik}/\partial z_{i0})},\ \forall\, i \in \boldsymbol{N},\ \forall\, c \in \boldsymbol{C}.$$

Remark 2: π_{ic} is characteristic c's marginal contribution to a person's *marginal happiness* through his/her functionings in terms of the numéraire attribute, z_{i0}. It can be interpreted also as a *hedonic marginal willingness-to-pay*(HMW), which corresponds to a "marginal rate of substitution between characteristic c and the numéraire characteristic, z_{i0}" in the utility theoretical context. π_{ic} can take whatever sign $\{+, 0, -\}$ from the above discussion. Remark that my "MRS" is different from Drèze and Hagen(1978), because our concept involves the functionings *à la* Sen. Hence, I must have replaced a happiness function with a utility function which cannot fully capture the roles of attributes and functionings.

The Gorman-Lancasterian characteristics theory is most suitable to analyze goods which are perfectly divisible and decomposable into characteristics. Equation (2) may be applied to any consumer whose utilizations, however, differ from person to person. Consequently, I have had to introduce each person's functionings as one of Sen's concepts to fully appraise the value of goods or characteristics. Each person's features differ, so I must have introduced the *characteristics availability function* represented by Eq.(2).

One of the issues is that in order the maximal value H_i^* to be chosen, then what is an individually optimal composition of attributes, z_i^*, embodied in the goods that he/she consumes, which corresponds to the value with the highest ranking measured by his/her happiness function. It can be said that the issue is to find an *individually optimal attributes mix*, z_i^*, such that $H_i^* = H_i(f_{i1}(z_i^*),..., f_{iK_i}(z_i^*))$ for some $b_i \in \boldsymbol{B}_i(\boldsymbol{X}_i)$. f_{ik} is a continuous function of a composition of characteristics, z_i. One may interpret b_i as a *Well-Being Index*, since b_i, *ceteris paribus*, corresponds to some level of well-being. In our context, a person enjoys his/her happiness, which can be enhanced by his/her utilization of functionings.

2.4. *Optimizations by Happiness Maximizing Consumers and Profit Maximizing Producers*

Here, I present the optimizations by profit maximizing producers who are to supply goods with Pareto optimal product quality to consumers to maximize their happiness functions. Each consumer has to solve his/her optimization problem and the maximand is his/her happiness function:

(8) $\quad Max \quad H_i = H_i(b_i)$

(9) $\quad b_i = (f_{i1}(z_i),..., f_{iK_i}(z_i)) \in \boldsymbol{B}_i(\boldsymbol{X}_i),$

(10) $$z_i = (z_{i0}, z_{i1}, \dots, z_{iC})$$

(11) $$z_{ic} = \sum_{j=1}^{J} x_{ij} q_{jc}, \quad \forall c \in \boldsymbol{C}$$

(12) $$\sum_{j=1}^{J} p_j x_{ij} = z_{i0}$$

(13) $$x_{ij} \geqq 0, \quad \forall j \in \boldsymbol{J}.$$

Then, we have the first result.

PROPOSITION 1. *An individually optimal consumption of goods as a composition of Gorman-Lancasterian attributes in terms of his/her functionings is characterized by the conditions:*

(14) $$\sum_{c=1}^{C} \pi_{ic} q_{jc} \leqq p_j, \ \left(\sum_{c=1}^{C} \pi_{ic} q_{jc} - p_j \right) x_{ij} = 0, \quad \forall i \in \boldsymbol{N}, \ \forall j \in \boldsymbol{J}, \ c \neq j'.$$

Remark 3: The Kuhn-Tucker conditions for the above optimization problems give the proof. In **Proposition 1** the conditions are not only necessary but also sufficient according to the assumptions imposed on the functions. In the above equations, π_{ic} signifies an individual hedonic price of attribute c utilized by person i's functionings. The left-hand side of the first equation in (14) is the sum of values of components embodied in x_{ij} units of good j. The formulae(14) mean that the unit price of the good j is equal to the aggregate marginal contributions of ingredients generated by person i's numéraire attribute to his/her happiness. The conditions in Eq.(14) assure a Pareto optimality for each good's quantity and give a basis upon whether consumers choose to buy goods.

When $x_{ij} > 0$, $p_j = \sum_c \pi_{jc} q_{jc}$ from Eq.(14), then I have the profit-maximization problem with P_j being producer j's profit: the first term is the revenue and the second term is the cost[2)].

(15) $$Max \quad P_j = \sum_{i=1}^{N} x_{ij} \sum_{c=1}^{C} \pi_{ic} q_{jc} - x_{j0}.$$

Thus, I have the next proposition.

PROPOSITION 2. *Necessary conditions for Pareto optimal product quality in terms of Gorman-Lancasterian attributes are:* $\forall j \in \boldsymbol{J},\ \forall c \in \boldsymbol{C},\ c \neq j'$

$$\sum_{i=1}^{N} \pi_{ic} x_{ij} \leqq \partial x_{j0}/\partial q_{jc},\ \left(\sum_{i=1}^{N} \pi_{ic} x_{ij} - \partial x_{j0}/\partial q_{jc}\right) q_{jc} = 0. \tag{16}$$

Remark 4: The formulae(16) are a natural extention of the conditions shown in Corollary 5.1 of Drèze and Hagen(1978) with a replacement of their MRSs by the modified form as advocated above. The left-hand side of the first equation in (16) is a marginal social value, and the right-hand side is the marginal cost of attribute c. The equations(16) may be referred to as the "Samuelsonian Hedonic Conditions", since quality attributes, q_{jc}, $\forall j \in \boldsymbol{J}$, $\forall c \in \boldsymbol{C}$, can be regarded as public goods and thus are common to all consumers. Consequently, Eqs.(16) establish a Pareto optimality for an amount of each attribute, and determine an optimal composition of characteristics, q_{jc}, $\forall j \in \boldsymbol{J}$, $\forall c \in \boldsymbol{C}$. Hence, **Propositions 1** and **2** verify respectively the quantitative and qualitative efficiency.

3. THE HEDONIC MDP PROCEDURES

3.1. *Definitions*

Denote an allocation as $z = (z_1, \ldots, z_N)$ and the set of feasible allocations as $\boldsymbol{Z}$.

Definition 1. An allocation z is feasible if it satisfies the following conditions:

$$y_j \in \boldsymbol{Y}_j,\ \forall j \in \boldsymbol{J} \tag{17}$$

$$z_i \in \boldsymbol{Z}_i,\ \forall i \in \boldsymbol{N} \tag{18}$$

$$\sum_{j=1}^{J} x_{j0} + \sum_{i=1}^{N} \sum_{c=1}^{C} z_{ic} \leqq \sum_{i=1}^{N} z_{i0} \tag{19}$$

$$\sum_{j=1}^{J} x_{ij} \leqq x_j,\ \forall j \in \boldsymbol{J} \tag{20}$$

$$z_{ic} \leqq \sum_{j=1}^{J} x_{ij} q_{jc} + \omega_{ic},\ \forall i \in \boldsymbol{N},\ \forall c \in \boldsymbol{C}. \tag{21}$$

Equations(17) and (18) are technical feasibility in production and consumption. Eqs.(19) and (21) are availability in input and output. Equation(20) means that all goods are private goods, whose prices and quantities are assumed to be optimally determined in the markets.

Definition 2. An allocation z is individually rational if and only if

(22) $$(\forall i \in \boldsymbol{N})\ [H_i(b_i(z_i)) \geqq H_i(b_i(\omega_i))].$$

Definition 3. A Pareto optimum for this economy is an allocation $z^* \in \boldsymbol{Z}$ such that there exists no feasible allocation z with

(23) $$(\forall i \in \boldsymbol{N})\ [H_i(b_i(z_i)) \geqq H_i(b_i(z_i{}^*))]$$

(24) $$(\exists j \in \boldsymbol{N})\ [H_j(b_j(z_j)) > H_j(b_j(z_j{}^*))].$$

3.2. *Samuelsonian Hedonic Conditions*

Denote a *hedonic marginal willingness to pay* as $\pi_{ijc} \equiv \pi_{ic} x_{ij}$ and a marginal cost as $\gamma_{jc} \equiv \partial x_{j0}/\partial q_{jc}$, $\forall j \in \boldsymbol{J}$, $\forall c \in \boldsymbol{C}$, $c \neq j'$, to define the Samuelsonian Hedonic Conditions based on **Proposition 2**.

Definition 4. ***Samuelsonian Hedonic Conditions*** (***SHC***) are denoted as:

(25) $$(\forall j \in \boldsymbol{J})\,(\forall c \in \boldsymbol{C})\ \left[\sum_{i=1}^{N} \pi_{ijc} \leqq \gamma_{jc} \text{ and } \left(\sum_{i=1}^{N} \pi_{ijc} - \gamma_{jc}\right) q_{jc} = 0\right].$$

Remark 5: SHC is reminiscent of the original Samuelson's Conditions which seem to have tacitly assumed public goods with fixed quality, i. e., $(q_{j1},\ldots, q_{jC})$ was supposed to be a vector of some constants, or it can be interpreted as the quality of public goods being already determined otherwise.

Our procedures presented below can attain both qualitative and quantitative Pareto optimality. Let $\mathbf{P}$, $\mathbf{P}_0$, and $\mathbf{B}$ be the sets of Pareto, individually rational Pareto, and boundary optima, respectively. I assume $\mathbf{P}_0 \cap \mathbf{B} = \phi$ so that the issue is the same as one in which the boundary optima has been elaborately avoided. To reach a point in $\mathbf{P} \backslash \mathbf{P}_0$ is not a task given to our procedures, so that I may confine myself to focus on the set $\mathbf{P}_0$. In order to achieve any limit point in $\mathbf{P}_0 \cap \mathbf{B} \neq \phi$, an alternative approach is needed [See Sato(2003)]. Conventional mathematical notation is used throughout in the same manner as in Sato(1983). Hereafter, all variables are assumed to be functions of time, however, the argument t is omitted unless confusion could arise.

3.3. *The ζ MDP Procedures*

The design of the planning procedures with public goods might be said to have fully developed to reach the acme in 1983. Initiated by three great pioneers—Edmond Malinvaud, and Jacques Drèze and Dominique de la Vallée Poussin—this field of research has made remarkable progress in the last three decades[3]. The analysis of incentives in planning procedures with public goods began in the late sixties and was mathematically refined by the characterization theorems of Champsaur and Rochet (1983), theorems that furthermore generalized the previous results of Fujigaki and Sato(1981), (1982), as well as Laffont and Maskin(1983). Most of these procedures can be characterized by the set of axioms: (i) Feasibility, (ii) Monotonicity, (iii) Pareto Efficiency, and (iv) Local Strategy Proofness. Formal definitions of these and three more conditions are given for our procedures in this section.

Introducing variable qualities lead firms to make quality/quantity for a fixed munber of goods. The insight presented is that siuce quality is the same for all consumers, it becomes a public good, so that we have to adopt planning procedures to set qualities.

The procedure I am designing belongs to the family of quantity-guided processes. The MDP Procedure is the best-known member belonging to the family of quantity-guided procedures, in which the planning center asks individual agents their MRSs between each public good and a private numéraire. Then the center revises the allocation according to the reported MRSs. The relevant information exchanged between the center and the periphery is in the form of quantity.

Let us use the following notation: $dq_{jc}(t)/dt \equiv Q_{jc}(t)$, $dz_{ic}(t)/dt \equiv Z_{ic}(t)$ and $dz_{i0}(t)/dt \equiv Z_{i0}(t)$. I employ a dynamic representation of the Samuelsonian hedonic conditions verified in **Proposition 2**. For any $t \in [0, \infty]$, the procedure for adjusting quality attributes embedded in goods when assuming truthful revelation of hedonic marginal willing-ness-to-pay reads:

$$(26) \qquad Q_{jc}(t) = \begin{cases} \sum_{i=1}^{N} \pi_{ijc}(t) - \gamma_{jc}(t) \\ Max\left\{0, \sum_{i=1}^{N} \pi_{ijc}(t) - \gamma_{jc}(t)\right\} \end{cases} \qquad \forall j \in \boldsymbol{J},\ \forall c \in \boldsymbol{C}$$

$$(27) \qquad Z_{ic}(t) = \sum_{j=1}^{J} x_{ij}(t)\, Q_{jc}(t),\ \forall i \in \boldsymbol{N},\ \forall c \in \boldsymbol{C}$$

$$(28) \qquad Z_{i0}(t) = \sum_{j=1}^{J} \sum_{c=1}^{C} \left\{ -\pi_{ijc}(t) + \delta_i \Big(\sum_{i=1}^{N} \pi_{ijc}(t) - \gamma_{jc}(t) \Big) \right\} Q_{jc}(t),\ \forall i \in \boldsymbol{N}$$

where $\delta_i > 0,\ \forall i \in \boldsymbol{N}$, and $\sum_{i=1}^{N} \delta_i = 1$.

Remark 6: $Q_{jc}(t)$ is the time derivative of attribute c's amount in one unit of good j at iteration t of the procedure. The Max operator enables us to avoid a reduction of an attribute's amount in the negative direction. $Z_{ic}(t)$ is a speed of adjustment in a quantity of a characteristic c in x_{ij} units of goods at time t. $Z_{i0}(t)$ is a revision in the numéraire attribute as the consumption of attributes evolve. $\delta = (\delta_1, \ldots, \delta_N)$ is a vector of distributional coefficients determined by the planner prior to the beginning of the operation of the procedure. Its role is to share among individuals the "social surplus", $\sum_j \sum_c \{Q_{jc}(t)\}^2$, which is always positive except in equilibrium. $\delta_i > 0$ was posited by Drèze and de la Vallée Poussin(1971), and followed by Roberts(1979), whereas $\delta_i \geqq 0$ was assumed by Champsaur(1976) who advocated a notion of neutrality to be explained below.

The dynamical system [(26), (27), (28)] is called the ζ *MDP Procedures* for the optimal provision of attributes, which preserve the desirable properties that the original MDP Process can enjoy; namely, feasibility, monotonicity, stability, and neutrality. Also the incentive properties pertaining to minimax and Nash equilibrium strategies were proved by Drèze and de la Vallée Poussin(1971), Henry(1979) and Roberts(1979).

Let us compare the original MDP and our ζMDP Procedures. The MDP Process evolves in the allocation space and stops when the Samuelson's condition is met so that the public good quantity is efficient, and the private good is simultaneously allocated in a Pareto optimal way. The ζMDP Procedures generate in the allocation space of attributes and stops when the Samuelsonian hedonic conditions hold; i. e., an allocation at that point is Pareto optimal in the generalized sense that quality of goods can vary. In the ζMDP Procedures the planning center must acquire $\pi_{ijc}(t)$ as a relevant information. Since $\pi_{ijc}(t)$ is private information about the marginal evaluation of the quality of goods that an individual consumes, the incentive problem could arise. One can decentralize the quality adjustment of each good, i. e., $Q_{jc}(t)$ can be made adjusted by each producer, however, technical information about the production, i. e., $\gamma_{jc}(t)$, is assumed to be known to the planner.

3.4. *The Local Incentive Game Along the ζ MDP Procedures*

For many years since the appearance of the seminal paper by Samuelson(1954), there prevailed a gloomy pessimism that the free rider problem was inevitable in the provision of public goods. This skeptical unanimity, however, was swept away by the accumulated literature on the sophisticated incentive compatible planning processes for providing public goods. The last three decades have witnessed numerous attempts to resolve the free rider problem or the problem of incentives by designing iterative planning procedures to efficiently supply public goods. Typically, these procedures involve asking participants to provide information on their preferences to a planning center in charge of allocating resources among individual agents.

The incentive problem associated with planning procedures to supply public goods may be summarized as follows: there exists a possibility that the agents participating in a process might have an incentive to purposely misstate their private information about their preferences with the hope of distorting the outcome that the procedure yields. The fundamental problem is how to elicit the unkown and unobservable information that is necessary to implement the planning rules.

The idea of employing game-theoretic notions in solving the incentive problem along planning paths of the procedures was first formally introduced into the literature by Drèze and de la Vallée Poussin(1971). Their procedure, which converges monotonically to an individually rational Pareto optimum, assures that true revelation of preferences for public goods is a minimax strategy for each individual. As for modeling incentives of the players, however, a minimax strategy is weaker and less attractive than the Nash strategy, Roberts(1979) and Henry(1979) studied the incentive properties along the solution paths of the MDP Process, by substituting myopic Nash behavior for minimax behavior at each iteration. Roberts(1979) verified that the process can achieve an individually rational Pareto optimum even under incorrect revelation, however, that the convergence speed slows down as participants in the procedure increases. Henry(1979) then refined Roberts' results on the incentive properties in the MDP Procedure by restricting individuals to report nonnegative messages.

Subsequently, Fujigaki and Sato(1981) and (1982) designed a satisfactory planning procedure generalizing the MDP Process, which assures that truthfulness is a dominant strategy for each player. It is a procedure which simultaneously converges monotonically to the unique individually rational Pareto optimal allocation. They also proved that any quantity-guided continuous planning procedure, fulfilling the normative conditions is characterized by the very procedure they established. The distributional implications of this characterization were also deduced.

Let us examine the incentive properties of our procedures in this subsection. Now the assumption of truthful revelation of preferences is relaxed. Each agent's announcement, ϕ_{ijc}, is not necessarily equal to his/her true hedonic marginal willingness-to-pay, π_{ijc}. A local incentive game associated with each iteration of the process is formally defined as the normal form game $(\boldsymbol{N}, \Psi, U)$; $\boldsymbol{N}$ is the set of players, $\Psi = \times_{i \in \boldsymbol{N}} \Psi_i \subset \boldsymbol{R}_+$ is the Cartesian product of Ψ_i which is player i's strategy set, and $U = (U_1, \ldots, U_{\boldsymbol{N}})$ is the N-tuple of payoff functions. The time derivative of individual i's happiness in the local incentive game played along the procedure is given by differentiating Eq.(6)

$$dH_i(t)/dt = \sum_{c=1}^{C} H_{ic} Z_{ic}(t) + H_{i0} Z_{i0}(t) \tag{29}$$

where $H_{ic} \equiv \sum_k (\partial H_i / \partial f_{ik})(\partial f_{ik} / \partial z_{ic})$ and $H_{i0} \equiv \sum (\partial H_i / \partial f_{ik})(\partial f_{ik} / \partial z_{i0})$.

Eq.(29) is proportional to the payoff for any player given by

$$U_i(t) = \sum_{j=1}^{J} \sum_{c=1}^{C} \pi_{ijc}(t)\, Q_{jc}(t) + Z_{i0}(t). \tag{30}$$

The behavioral hypothesis underlying the above equations is the following assumption of myopia: i. e., in order to maximize his/her instantaneous happiness increment, $U_i(t)$, each player determines his/her dominant strategy, $\phi_{ijc}{}^* \in \Psi_i$ such that

$$(\forall \phi \in \Psi)(\forall \phi_i \in \Psi_i)(\forall i \in N) \left[\sum_{j=1}^{J} \sum_{c=1}^{C} \pi_{ijc} Q_{jc}(\phi_{ijc}{}^*, \phi_{-ijc}) + Z_{i0}(\phi_i{}^*, \phi_{-i}) \geqq \sum_{j=1}^{J} \sum_{c=1}^{C} \pi_{ijc} Q_{jc}(\phi_{ijc}, \phi_{-ijc}) + Z_{i0}(\phi_i, \phi_{-i}) \right] \tag{31}$$

where

$\phi = (\phi_1, \ldots, \phi_N)$, $\phi_i = (\phi_{i11}, \ldots, \phi_{iCJ})$, $\phi_{-ijc} = (\phi_{1jc}, \ldots, \phi_{i-1,jc}, \phi_{i+1,jc}, \ldots, \phi_{Njc})$, $\phi_i{}^* = (\phi_{i11}{}^*, \ldots, \phi_{iCJ}{}^*)$, and $\phi_{-i} = (\phi_1, \ldots, \phi_{i-1}, \phi_{i+1}, \ldots, \phi_N)$.

3.5. *Normative Conditions for the ζ MDP Procedures*

The conditions based on *SHC* are in order.

Condition ***F***. *Feasibility*:

$$(\forall \phi \in \Psi)(\forall t \in [0, \infty])\left[\sum_{i=1}^{N} Z_{i0}(t) + \sum_{j=1}^{J}\sum_{c=1}^{C} \gamma_{jc}(t) Q_{jc}(t) = 0\right].$$

Condition ***M***. *Monotonicity*:

$$(\forall \phi \in \Psi)(\forall i \in \boldsymbol{N})(\forall t \in [0, \infty))\left[U_i = \sum_{j=1}^{J}\sum_{c=1}^{C} \pi_{ijc}(t) Q_{jc}(t) + Z_{i0}(t) \geqq 0\right].$$

Condition ***PE***. *Pareto Efficiency*:

$$(\forall \phi \in \Psi)(\forall i \in \boldsymbol{J})(\forall c \in \boldsymbol{C})\left[Q_{jc} = 0 \ \Leftrightarrow \sum_{i=1}^{N} \phi_{ijc} = \gamma_{jc}\right].$$

Condition ***LSP***. *Local Strategy Proofness*:

$$(\forall \phi \in \Psi)(\forall \phi_i \in \Psi_i)(\forall i \in \boldsymbol{N})(\forall t \in [0, \infty])$$

$$\left[\sum_{j=1}^{J}\sum_{c=1}^{C} \pi_{ijc}(t) Q_{jc}(\pi_{ijc}(t), \phi_{-ijc}(t)) + Z_{i0}(\pi_i(t), \phi_{-i}(t)) \right.$$
$$\left. \geqq \sum_{j=1}^{J}\sum_{c=1}^{C} \pi_{ijc}(t) Q_{jc}(\phi_{ijc}(t), \phi_{-ijc}(t)) + Z_{i0}(\phi_i(t), \phi_{-i}(t))\right]$$

where $\pi_i(t) = (\pi_{i11}(t), \ldots, \pi_{iJC}(t))$.

Remark 7: Conditions except ***PE*** must be fulfilled for any $t \in [0, \infty]$. Champsaur and Rochet(1983) gave a systematic study on the family of quantity-guided planning procedures which are asymptotically efficient and locally strategy proof. These procedures were classified in Sato(1986). Sato(2003) introduced the class of the Generalized νMDP Procedures that enjoy all of the normative conditions for quantity-guided planning procedures and that they reach even the set of boundary optima. The original MDP Procedure does satisfy Condition ***LSP*** only in the two-person economy, though. Local strategy proofness is equivalent to *Strongly Locally Individually Incentive*

Compatibility. This paper is confined to ***LSP***, postponing the issue of *Coalitional Local Strategy Proofness*.

4. PROPERTIES OF THE ζMDP PROCEDURES

4.1. *Conditions **F**, **M**,* and ***PE***

Now let me examine the properties of the ζMDP Procedures just defined above. Condition ***F*** is easily checked to be satisfied, since it has been already used to formulate the ζMDP Procedures. Condition ***M*** is verified under correct revelation as follows. This is simply derived from the fact that

$$U_i(t) = \sum_{c=1}^{C} \pi_{ijc}(t) Q_{jc}(t) + Z_{i0}(t) = \delta_i \sum_{j=1}^{J} \sum_{c=1}^{C} (Q_{jc}(t))^2 \geqq 0. \tag{32}$$

Thus, I have the following:

Theorem 1. *The ζMDP Procedure satisfies Condition **M** for $\delta_i > 0$, $\forall i \in \boldsymbol{N}$.*

Condition ***PE*** comes from ***SHC*** and the former should be regarded as one of the desiderata that the procedure has to possess.

4.2. *Minimax and Nash Equilibrium Strategies*

What about the incentive properties of the ζMDP Procedures? The results are the same as with the original MDP Process, as seen below.

i) *Minimax strategy*

For this concept, the following theorem holds.

Theorem 2. *Revealing π_{ijc} truthfully in the ζMDP Procedures is a minimax strategy for any $i \in \boldsymbol{N}$. It is the only minimax strategy for any $i \in \boldsymbol{N}$, when $q_{jc} > 0$ for any $j \in \boldsymbol{J}$ and for any $c \in \boldsymbol{C}$.*

Proof: In order to prove this theorem, let us modify our procedure for any $t \in [0, \infty]$:

$$Q_{jc}(t) = \xi_{jc}(t) \left\{ \sum_{i=1}^{N} \phi_{ijc}(t) - \gamma_{jc}(t) \right\}, \ \forall j \in \boldsymbol{J}, \ \forall c \in \boldsymbol{C} \tag{33}$$

(34) $$\xi_{jc}(t) = \begin{cases} 0 & \text{if } q_{jc}(t) = 0 \text{ and } Q_{jc}(t) < 0, \\ 1 & \text{otherwise,} \end{cases} \quad \forall j \in \boldsymbol{J}, \ \forall c \in \boldsymbol{C}$$

(35) $$Z_{ic}(t) = \sum_{j=1}^{J} x_{ij}(t)\, Q_{jc}(t), \ \forall i \in \boldsymbol{N}, \forall c \in \boldsymbol{C}$$

(36) $$Z_{i0}(t) = \sum_{j=1}^{J} \sum_{c=1}^{C} Q_{jc}(t) \{\delta_i Q_{jc}(t) - \phi_{ijc}(t)\}, \ \forall i \in \boldsymbol{N}$$

where $\delta_i > 0$, $\forall i \in \boldsymbol{N}$, and $\sum_{i=1}^{N} \delta_i = 1$. The parameter, $\xi_{jc}(t)$, is used here to avoid the negative adjustment if $q_{jc}(t) = 0$ and $Q_{jc}(t) < 0$, $\forall j \in \boldsymbol{J}$, $\forall c \in \boldsymbol{C}$. This is due to Drèze and de la Vallée Poussin(1971). Temporarily omit the argument t.

Each agent aims at minimizing his opponent's payoff, then one observes

(37) $$\partial U_i(\phi)/\partial \phi_{hjc} = \sum_{j=1}^{J} \xi_{jc} \left\{ (\pi_{ijc} - \phi_{ijc}) + 2\delta_i \left(\sum_{i \neq h} \phi_{ijc} + \phi_{hjc} - \gamma_{jc} \right) \right\} = 0.$$

Thus, we have

(38) $$\phi_{hjc} = (\pi_{ijc} - \phi_{ijc})/2\delta_i + \sum_{i \neq h} \phi_{ijc} - \gamma_{jc}$$

When the agents $h \neq i$, $\forall h \in \boldsymbol{N}$, use this strategy, the payoff to agent i is obtained as

(39) $$U_i(\phi) = -\sum_{j=1}^{J} \sum_{c=1}^{C} \xi_{jc} (\pi_{ijc} - \phi_{ijc})^2 / 4\delta_i \leqq 0.$$

Hence, only $\pi_{ijc} = \phi_{ijc}$, $\forall c \in \boldsymbol{C}$, $\forall j \in \boldsymbol{J}$, i. e., correct revelation assures $U_i(\phi)$ to be maximized irrespective of the strategies followed by the others. Q.E.D.

ii) *Nash equilibrium strategy*

Next, I examine the Nash property of the ζMDP Procedures. By using the results of Roberts(1979), I obtain the unique Nash equilibrium strategy as follows:

(40) $$\phi_{ijc} = \pi_{ijc} - \frac{1 - 2\delta_i}{N-1} \left(\sum_{i=1}^{N} \phi_{ijc} - \gamma_{jc} \right), \ \forall i \in \boldsymbol{N}, \ \forall j \in \boldsymbol{J}, \ \forall c \in \boldsymbol{C}.$$

In general, $\phi_{ijc} \neq \pi_{ijc}$ follows for $\forall i \in \boldsymbol{N}$, $\forall j \in \boldsymbol{J}$, and $\forall c \in \boldsymbol{C}$, since $\sum_i \phi_{ijc} - \gamma_{jc}$ is not necessarily zero for any $j \in \boldsymbol{J}$ and for any $c \in \boldsymbol{C}$. However, $\sum_i \phi_{ijc} - \gamma_{jc} = 0$ holds,

$\forall c \in \boldsymbol{C}$, $\forall j \in \boldsymbol{J}$, at the equilibrium, hence, $\phi_{ijc} = \pi_{ijc}$, $\forall j \in \boldsymbol{J}$, $\forall c \in \boldsymbol{C}$, results for each individual. Thus, the theorem follows.

Theorem 3. *$\phi_{ijc} = \pi_{ijc}$ holds for $\forall i \in \boldsymbol{N}$, $\forall j \in \boldsymbol{J}$,* and *$\forall c \in \boldsymbol{C}$, at the equilibrium of the ζMDP Procedures.*

4.3. *Neutrality of the ζMDP Procedures*

It was Champsaur(1976) who first advocated the notion of neutrality for the MDP Procedure, and Cornet(1983) generalized it by omitting two restrictive assumptions imposed by Champsaur, i. e., (i) uniqueness of solution and (ii) concavity of the utility functions. Neutrality depends on the distributional coefficient vector δ. Remember that its role is to attain any individually rational Pareto optima(IRPO) by distributing the social surplus generated during the operation of the procedure: δ varies the trajectory to reach every IRPO. In other words, the center can guide allocations via the choice of δ, however, it cannot predetermine any final allocation to be reached.

Condition $\boldsymbol{N}$. *Neutrality*:

For every efficient point $z^* \in \boldsymbol{Z}$ and an initial point $z_0 \in \boldsymbol{Z}$, there exist δ and $z(t, \delta)$, a trajectory starting at z_0, such that $z^* = z(\infty, \delta)$.

Remark 8: (i) Neutrality is autonomous, while Champsaur and Rochet's(1983) local neutrality is not autonomous. For the other concepts of neutrality associated with planning procedures, see Sato(1983) and (1986). D'Aspremont and Drèze(1979) advocated an alternative version of neutrality which is valid for the general context. See also the proofs given by Cornet(1977a, b), and Cornet and Lasry(1976).

(ii) The crucial underpinning of Champsaur-Cornet's neutrality is the nonnegativity requirement of δ. Once dropping this, one cannot prove their neutrality theorems. Originally, Drèze and de la Vallée Poussin(1971) imposed the hypothesis of positive δ: with this assumption, they could demonstrate their **Theorem 3** on minimax strategy. Successors except Roberts(1979) imposed the nonnegativity of δ to obtain some fruitful results.

Clearly, the original MDP Procedure for quantity adjustment satisfies Condition

$\boldsymbol{N}$ which, however, assumed interior optima and nonnegative δ. In order to be able to reach the Samuelsonian hedonic optima, let me apply Champsaur-Cornet's neutrality theorem.

Theorem 4. *Under Assumptions 1〜3, for every individually rational Pareto optimum* z^*, *there exist* δ *and a solution path,* $z(\cdot):[0, \infty) \to \boldsymbol{Z}$ *of the dynamical system defining the* ζ *MDP Procedures such that,* $H_i(b_i(z_i^*)) = lim_{t\to\infty} H_i(b_i(z_i(t)))$, $\forall i \in \boldsymbol{N}$.

Proof: Thanks to Cornet's neutrality theorem (1983, **Theorem 5.2**), the original MDP Procedure is neutral. Now we know the ζ MDP Procedures which can also attain **SHC**, thus the differential equations must be substituted for the ζ MDP Process in the proof to Cornet's neutrality theorem. The assumptions that Cornet imposed are easily checked to be fulfilled in our context. Q.E.D.

4.4. *Existence and Stability of Solutions*

This subsection considers the issues on the existence and stability of the solutions reached by the ζ MDP Procedures and I have the following theorem.

Theorem 5. *For the* ζ *MDP Procedures and for* $z_0 \in \boldsymbol{Z}$, *there exists a unique trajectory* $z(\cdot):[0, \infty) \to \boldsymbol{Z}$, *which is such that* $lim_{t\to\infty} z(t)$ *exists and is a Pareto optimum.*

Proof: (i) Existence.

It is clear that the differential equations defining our ζMDP Procedures given by Eqs.(26)-(28) are not locally Lipschitzian continuous, I must therefore deal with possible discontinuity. In order to verify the existence of solutions to our procedure, we must modify it. However, $Q_{jc}(t)$, $Z_{ic}(t)$, and $Z_{i0}(t)$, defined by Equations(33)-(36) are all Lipschitzian, so that starting from such a point z_0 there exists a locally unique solution path $z(t, z_0)$, by **Theorem 1** in Hirsch and Smale(1974, Ch. 15). Discontinuity has been, as it were, included or "internalized" in the parameter, $\xi_{jc}(t)$.

(ii) Convergence.

To prove the second part of the Theorem, recollect an immediate corollary to **Theorem 6.1** in Champsaur, Drèze and Henry(1977). If a dynamic system has a unique solution for any $z_0 \in \boldsymbol{Z}$, say $z(t, z_0)$, which varies continuously with z_0

remains in $\boldsymbol{Z}$ for all t, and if there is a Lyapunov function, then the system is quasi-stable.

As a function pertinently chosen for a Lyapunov function, let us take the weighted sum of the agents' happiness functions:

$$(41) \qquad L(z(t)) = \sum_{i=1}^{N} \lambda_i H_i(t)$$

where λ_i is assumed to be an inverse of H_{i0}. Differentiating Eq.(41) with respect to time gives

$$(42) \qquad dL(z(t))/dt = \sum_{i=1}^{N} \lambda_i (dH_i/dt) = \delta_i \sum_{j=1}^{J} \sum_{c=1}^{C} \left(\sum_{i=1}^{N} \psi_{ijc} - \gamma_{jc} \right)^2 \geqq 0.$$

Clearly, any equilibrium of the ζMDP Procedures is a Pareto efficient allocation, one only has to verify that $lim_{t \to \infty} z(t, z_0)$ exists for any $z_0 \in \boldsymbol{Z}$. This is immediate from our convexity assumptions, there is only one Pareto optimum z^* determined by $L(z^*) = lim_{t \to \infty} L(z(t))$. Q.E.D.

4.5. *Nonlinearizing the ζ MDP Procedures*

Our accumulated knowledge of incentives can be used to nonlinearize our ζ MDP Procedures in such a parallel manner as in Fujigaki and Sato(1981).

The Nonlinearized ζ MDP Procedure reads for any $t \in [0, \infty)$:

$$(43) \qquad Q_{jc}(t) = \xi_{jc}(t) \left\{ \sum_{i=1}^{N} \pi_{ijc}(t) - \gamma_{jc}(t) \right\} \left| \sum_{i=1}^{N} \pi_{ijc}(t) - \gamma_{jc}(t) \right|^{N-2}, \quad \forall j \in \boldsymbol{J}, \ \forall c \in \boldsymbol{C}$$

$$(44) \qquad \xi_{jc}(t) = \begin{cases} 0 & \text{if } q_{jc}(t) = 0 \text{ and } Q_{jc}(t) < 0, \\ 1 & \text{otherwise,} \end{cases} \qquad \forall j \in \boldsymbol{J}, \ \forall c \in \boldsymbol{C}$$

$$(45) \qquad Z_{ic}(t) = \sum_{j=1}^{J} x_{ij}(t)\, Q_{jc}(t), \ \ \forall i \in \boldsymbol{N}, \ \forall c \in \boldsymbol{C}$$

$$(46) \qquad Z_{i0}(t) = \sum_{j=1}^{J} \sum_{c=1}^{C} Q_{jc}(t) \{(1/N) Q_{jc}(t) - \pi_{ijc}(t)\}, \ \ \forall i \in \boldsymbol{N}.$$

In our context, as one of the planner's tasks is to achieve an optimal composition of quality attributes in private goods, he/she has to collect the relevant information from the peripheral agents so as to meet the conditions presented above. Fortunately, the necessary information is available if the procedure is strongly locally individually incentive compatible. It was already shown by Fujigaki and Sato(1982) that the locally strategy proof Generalized MDP Procedure cannot preserve neutrality, since δ was concluded to be fixed; i. e., $1/N$, to achieve ***LSP***, keeping the other conditions fulfilled. Thus, we have the following result.

Theorem 6. *The Nonlinearized ζ MDP Procedure satisfies Conditions **F**, **M**, **PE**, and **LSP**. Conversely, any ζMDP Process satisfying these conditions is characterized to the Nonlinearized ζ MDP Procedure.*

Proof is postponed to Section 5.

Remark 9: In our papers(1981) and (1982), we labeled our procedure as the *Generalized MDP Procedure* for determining an optimal public good quantity. Certainly, its public good decision function was generalized or nonlinearized to include the MDP Procedure, however, the distributional vector was fixed to a specific value, i. e., $\delta_i = 1/N \neq 0$, $\forall i \in \boldsymbol{N}$, since N cannot be zero. Hence, the Nonlinearized ζ MDP Procedure may reach only the subset of the limit points. To be more precise, our process is called the *Nonlinearized ζ MDP Procedure*, which is a member belonging to the class of the Generalized ζ MDP Procedures. The *genuine* Generalized ζ MDP Procedures are presented below. For that purpose, I need the following conditions to be satisfied by the subclass of the ζ MDP Procedures.

4.6. *Aggregate Correct Revelation*

The operation of the Generalized MDP Procedure we proposed in (1981) does not even require truthfulness of each player to be a Nash equilibrium strategy, but it only needs an aggregate correct revelation to be a Nash equilibrium, as shown in Sato(1983). It is easily seen from the discussion in the previous subsections that the Nonlinearized ζ MDP Procedure is not neutral at all, which means that local strategy proofness impedes the attainment of neutrality. Hence, Sato(1983) proposed another version of neutrality, and Condition *Aggregate Correct Revelation* (***ACR***) which is weaker than ***LSP***. By ***ACR***, I can generalize the ζ MDP Procedures to which the

Nonlinearized ζMDP Procedure belongs. ***ACR*** can be stated in our context as follows:

*Condition **ACR**. Aggregate Correct Revelation*:

$$(\forall \pi(t) \in \mathbf{\Pi})(\forall j \in \boldsymbol{J})(\forall c \in \boldsymbol{C})(\forall t \in [0, \infty))\left[\sum_{i=1}^{N} \phi_{ijc}(\pi(t)) = \sum_{i=1}^{N} \pi_{ijc}(t)\right]$$

where $\pi(t) = (\pi_1(t), \ldots, \pi_N(t)) \in \mathbf{\Pi}$: the set of vectors, $\pi_i(t) = (\pi_{i11}(t), \ldots, \pi_{iJC}(t))$, and ϕ_{ijc} is defined in Eq.(40).

Remark 10: Condition ***ACR*** means that the sum of the Nash equilibrium strategies, ϕ_{ijc}, $\forall i \in \boldsymbol{N}$, always coincides with the aggregate value of the correct individual hedonic willingness-to-pay for the attributes. Clearly, ***ACR*** claims truthfulness only in the aggregate.

The following additional conditions are necessitated to acquire our desired results.

*Condition **TN**. Transfer Neutrality*:

$$\mathrm{O}(\forall z^* \in P_0)(\exists \tau \in \mathbf{\Delta})(\exists z(\cdot) \in \boldsymbol{Z})(\forall t \in [0, \infty))[z^* = \lim_{t\to\infty} z(t)]$$

where $\mathbf{\Delta}$ is the set of transfer rules: $\tau = \{\tau_1, \ldots, \tau_N\}$, and $z(\cdot)$ is a solution of the process.

*Condition **TA**. Transfer Anonimity*:

$$(\forall \pi(t) \in \mathbf{\Pi})(\forall i \in \boldsymbol{N})(\forall t \in [0, \infty))[\tau_i(\pi(t)) = \tau_i(\rho(\pi(t)))]$$

where $\rho : \boldsymbol{R}_+^N \to \boldsymbol{R}_+^N$ is a permutation function.

Remark 11: Condition ***TA*** says that agent i's transfer in numéraire is invariant under permutation of its arguments: i.e., the order of strategies does not affect the value of $\tau_i(\pi)$, $\forall i \in \boldsymbol{N}$.

Keeping the same nonlinear decision function of attributes as derived with Condition ***LSP***, I can state the characterization theorem.

Theorem 7. *The Generalized ξMDP Procedures fulfill Conditions **ACR**, **F**, **M**, **PE**, **TA** and **TN**. Conversely, any ξMDP Process satisfying these conditions is characterized to:*

$$(47)\qquad Q_{jc}(t) = \xi_{jc}(t)\left\{\sum_{i=1}^{N}\pi_{ijc}(t)-\gamma_{jc}(t)\right\}\left|\sum_{i=1}^{N}\pi_{ijc}(t)-\gamma_{jc}(t)\right|^{N-2}, \quad \forall j \in \boldsymbol{J},\ \forall c \in \boldsymbol{C}$$

$$(48)\qquad \xi_{jc}(t) = \begin{cases} 0 & \text{if } q_{jc}(t) = 0 \text{ and } Q_{jc}(t) < 0, \\ 1 & \text{otherwise,} \end{cases} \quad \forall j \in \boldsymbol{J},\ \forall c \in \boldsymbol{C}$$

$$(49)\qquad Z_{ic}(t) = \sum_{j=1}^{J} x_{ij}(t)\, Q_{jc}(t), \quad \forall i \in \boldsymbol{N},\ \forall c \in \boldsymbol{C}$$

$$(50)\qquad Z_{i0}(t) = -\sum_{j=1}^{J}\sum_{c=1}^{C} Q_{jc}(t)\pi_{ijc}(t)+\tau_i(t), \ \forall i \in \boldsymbol{N}.$$

Proof: See Section 5.

Chander(1993) proved the incompatibility between core convergence property and local strategy proofness. It is possible to escape from his "impossibility theorem" by weakening the incentive requirement from ***LSP*** to ***ACR***, hence I can present the following:

Corollary 1. *There exists a member in the class of the Generalized ξMDP Procedures whose solutions converge to the core.*

Proof: Obviously, the family of the Generalized *ξMDP* Procedures involves as its member the Process with

$$(51)\qquad \tau_i(t) = \delta_i(t)\sum_{j=1}^{J}\sum_{c=1}^{C}\left\{\sum_{i=1}^{N}\pi_{ijc}(t)-\gamma_{jc}(t)\right\}Q_{jc}(t)$$

where

$$(52)\qquad \delta_i(t) = \frac{\sum_c \pi_{ijc}(t)}{\sum_h \sum_c \pi_{hjc}(t)}.$$

Some calculation leads us to observe

$$(53)\qquad Z_{i0}(t) = -\frac{\sum_c \pi_{ijc}(t)}{\sum_h \sum_c \pi_{hjc}(t)}\sum_{j=1}^{J}\sum_{c=1}^{C}\left\{\sum_{j=1}^{J}\sum_{c=1}^{C}\pi_{ijc}(t)-\gamma_{jc}(t)\right\}Q_{jc}(t).$$

In light of **Theorem 3.4** in Chander (1993), the procedure with Eq. (53) which replaces Eq. (50) clearly belongs to the class of processes he proposed, whose solutions converge to the core. Q.E.D.

Almost all MDP-type planning procedures designed so far share a common property that a social surplus in numéraire appears at each iteration during the working of the process. This surplus is distributed among all individuals according to the distributional coefficients specified by a constant N-dimensional vector, δ. All these planning processes assume that this vector is determined exogenously by the planner and prior to the beginning of the procedure, without resorting to any knowledge about the periphery, which has often been criticized.

In this subsection, I attempt to internalize the choice of surplus sharing in procedures and tried to give a possible solution for endogenous determination of surplus sharing by specifying transfer function which has the role of distributing surplus. Representative candidates for internally determining δ would be as follows:

$$\delta_i = \sum\nolimits_c \pi_{ijc} / \sum\nolimits_h \sum\nolimits_c \pi_{hjc}, \tag{54}$$

and

$$\delta_i = \frac{\sum_{h \neq i} \sum_c \pi_{hjc}}{(N-1) \sum_h \sum_c \pi_{hjc}}. \tag{55}$$

(54) and (55) have obvious implications respectively; (55) signifies that the smaller π_{ic}, the larger δ_i, which may give agents an incentive to purposely misstate their hedonic marginal willingness-to-pay for attributes as public goods. Hence, I have chosen (54) to get the above core-convergence result. Next section shows the proofs to our main theorems.

5. PROOFS OF THEOREMS

5.1. *Proof to the **Theorem 6***

It is easy to see that Conditions ***F*** and ***M*** are satisfied, which entail ***PE***. As regards ***LSP***, I prove the **Theorem 6** by modifying the proof to the **Theorem 6** in Sato (2003) to our process. The argument t is omitted hereafter.

Consider the following procedure:

(56) $$Q_{jc} = \xi_{jc} G_{jc}(\theta_{jc}), \ \forall j \in \boldsymbol{J}, \ \forall c \in \boldsymbol{C}$$

(57) $$\xi_{jc} = \begin{cases} 0 & \text{if } q_{jc} = 0 \text{ and } Q_{jc} < 0, \\ 1 & \text{otherwise,} \end{cases} \quad \forall j \in \boldsymbol{J}, \ \forall c \in \boldsymbol{C}$$

(58) $$Z_{ic} = \sum_{j=1}^{J} x_{ij} Q_{jc}, \ \forall i \in \boldsymbol{N}, \ \forall c \in \boldsymbol{C}$$

(59) $$Z_{i0} = \sum_{j=1}^{J} \sum_{c=1}^{C} \xi_{jc} \{ -\phi_{ijc} Q_{jc} + \delta_i(\theta) \theta_{jc} G_{jc}(\theta_{jc}) \}, \ \forall i \in \boldsymbol{N}$$

where $\theta_{jc} = \sum_{i=1}^{N} \phi_{ijc} - \gamma_{jc}$, and $\theta = (\theta_{11}, \ldots, \theta_{JC})$.

With this procedure, differentiating the payoff $U_i(\phi)$ with respect to ϕ_{ijc} gives

(60) $$\frac{\partial U_i}{\partial \phi_{ijc}} = \sum_{j=1}^{J} \sum_{c=1}^{C} \xi_{jc} x_{ij} [-G_{jc}(\theta_{jc}) + \delta_i'(\theta) \theta_{jc} G_{jc}(\theta_{jc}) + \delta_i(\theta) \{ G_{jc}(\theta_{jc}) + \theta_{jc} G_{jc}'(\theta_{jc}) \}] = 0$$

which is a necessary condition for the truth-telling to be a dominant strategy for any player.

Since $\sum_j \sum_c \xi_{jc} x_{ij} \neq 0$, we have

(61) $$\delta_i'(\theta) + \delta_i(\theta) \sum_{j=1}^{J} \sum_{c=1}^{C} \left\{ (\theta_j)^{-1} + \frac{G_{jc}'(\theta_{jc})}{G_{jc}(\theta_{jc})} \right\} = \sum_{j=1}^{J} \sum_{c=1}^{C} (\theta_{jc})^{-1}$$

By the formula of inhomogeneous linear differential equations, we observe

(62) $$\delta_i(\theta) = \exp(-\Theta) \sum_{j=1}^{J} \sum_{c=1}^{C} \left\{ \int (\theta_{jc})^{-1} \exp(\Theta) d\theta_{jc} + D_i(\phi_{-ijc}) \right\}, \ \forall i \in \boldsymbol{N}$$

where $D_i(\phi_{-ijc})$ is a real valued function independent of ϕ_{-ijc}, and

(63) $$\Theta = \sum_{j=1}^{J} \sum_{c=1}^{C} \int \left\{ (\theta_{jc})^{-1} + \frac{G'(\theta_{jc})}{G(\theta_{jc})} \right\} d\theta_{jc}.$$

The equations

(64) $$\exp(-\Theta) = \left[\sum_{j=1}^{J}\sum_{c=1}^{C}\theta_{jc}G_{jc}(\theta_{jc})\right]^{-1} \equiv [\tau(\theta)]^{-1}$$

and

(65) $$\exp(\Theta) = \sum_{j=1}^{J}\sum_{c=1}^{C}\theta_{jc}G_{jc}(\theta_{jc}) \equiv \tau(\theta)$$

yield

(66) $$\delta_i(\theta) = [\tau(\theta)]^{-1}\sum_{j=1}^{J}\sum_{c=1}^{C}\left\{\int G_{jc}(\theta_{jc})d\theta_{jc}+D_i(\phi_{-ijc})\right\},\ \forall i \in \boldsymbol{N}.$$

Denoting $\delta_i(\theta)\tau(\theta) \equiv \tau_i(\theta)$, we have

(67) $$\tau_i(\theta) = \sum_{j=1}^{J}\sum_{c=1}^{C}\left\{\int G_{jc}(\theta_{jc})d\theta_{jc}+D_i(\phi_{-ijc})\right\},\ \forall i \in \boldsymbol{N}.$$

Rewriting (67) in a definite integral form gives

(68) $$\tau_i(\theta) = \sum_{j=1}^{J}\sum_{c=1}^{C}\left\{\int_{\gamma_{jc}-\sum_{h\neq i}\phi_{hic}}^{\phi_{ijc}} G_{jc}(\phi_{ijc},\sum_{h\neq i}\phi_{hjc}-\gamma_{jc})d\theta_{jc}+D_i(\phi_{-ijc})\right\},\ \forall i \in \boldsymbol{N}.$$

which can be written as

(69) $$\tau_i(\theta) = \sum_{j=1}^{J}\sum_{c=1}^{C}\left\{\int_{0}^{\phi_{ijc}} G_{jc}(\theta_{jc})d\theta_{jc}+D_i(\phi_{-ijc})\right\},\ \forall i \in \boldsymbol{N}.$$

Letting $\theta_{jc} = 0, \forall j \in \boldsymbol{J}, \forall c \in \boldsymbol{C}$, then we get

(70) $$\tau_i(0) = D_i(\phi_{-ijc}),\ \forall i \in \boldsymbol{N}.$$

But, from Conditions $\boldsymbol{F}$ and $\boldsymbol{M}$, we have

(71) $$\tau_i(0) = 0,\ \forall i \in \boldsymbol{N}$$

which implies that

(72) $$D_i(\phi_{-ijc}) = 0.$$

Consequently, we obtain

$$(73)\qquad \tau_i(\theta) = \sum_{j=1}^{J}\sum_{c=1}^{C}\int_0^{\theta_{jc}} G_{jc}(\mu_{jc})\,d\mu_{jc},\ \ \forall\, i \in \boldsymbol{N}.$$

I observe therefore

$$(74)\qquad \sum_{j=1}^{J}\sum_{c=1}^{C}\theta_{jc}G_{jc}(\theta_{jc}) = \sum_{i=1}^{N}\tau_i(\theta) = N\sum_{j=1}^{J}\sum_{c=1}^{C}\int_0^{\theta_{jc}} G_{jc}(\mu_{jc})\,d\mu_{jc}.$$

Differentiating (74) with respect to θ_{jc} yields

$$(75)\qquad G_{jc}(\theta_{jc})+\theta_{jc}dG_{jc}(\theta_{jc})/d\theta_{jc} = NG_{jc}(\theta_{jc}).$$

Thus, we have

$$(76)\qquad \frac{dG_{jc}(\theta_{jc})/d\theta_{jc}}{G_{jc}(\theta_{jc})} = \frac{N-1}{\theta_{jc}},\ \ \forall\, j \in \boldsymbol{J},\ \ \forall\, c \in \boldsymbol{C}.$$

Solving (76) for $G_{jc}(\theta_{jc})$, we obtain

$$(77)\qquad G_{jc}(\theta_{jc}) = (\theta_{jc})^{N-1},\ \ \forall\, j \in \boldsymbol{J},\ \ \forall\, c \in \boldsymbol{C}.$$

Since $G_{jc}(\theta_{jc})$ is proved to be sign-preserving from Conditions $\boldsymbol{F}$ and $\boldsymbol{M}$, finally we get

$$(78)\qquad G_{jc}(\theta_{jc}) = \theta_{jc}|\theta_{jc}|^{N-2},\ \ \forall\, j \in \boldsymbol{J},\ \ \forall\, c \in \boldsymbol{C}.$$

As can be easily seen from (74)

$$(79)\qquad \tau_1 = \tau_2 = \cdots = \tau_N.$$

which reduces to

$$(80)\qquad \sum_{j=1}^{J}\sum_{c=1}^{C}\theta_{jc}G_{jc}(\theta_{jc}) \equiv N\tau_i.$$

Hence, we can conclude that

$$(81)\qquad \tau_i = (1/N)\sum_{j=1}^{J}\sum_{c=1}^{C}\theta_{jc}G_{jc}(\theta_{jc}),\ \ \forall\, i \in \boldsymbol{N}. \qquad \text{Q.E.D.}$$

5.2. *Proof to the* ***Theorem 7***

The proof to the **Theorem 1** in Sato(1983) where he proved for a public good can be applied because quality attributes are regarded as public goods.

Consider the following procedure:

(82) $$Q_{jc} = \xi_{jc} G_{jc}(\theta_{jc}),\ \forall j \in \boldsymbol{J},\ \forall c \in \boldsymbol{C}$$

(83) $$\xi_{jc} = \begin{cases} 0 \text{ if } q_{jc} = 0 \text{ and } Q_{jc} < 0, \\ 1 \text{ otherwise,} \end{cases} \quad \forall j \in \boldsymbol{J},\ \forall c \in \boldsymbol{C}$$

(84) $$Z_{ic} = \sum_{j=1}^{J} x_{ij} Q_{jc},\ \forall i \in \boldsymbol{N},\ \forall c \in \boldsymbol{C}$$

(85) $$Z_{i0} = \sum_{j=1}^{J} \sum_{c=1}^{C} \phi_{ijc} Q_{jc} + \tau_i(\theta),\ \forall i \in \boldsymbol{N}$$

where $\theta_{jc} \sum_{i=1}^{N} \phi_{ijc} - \gamma_{jc}$, and $\theta = (\theta_{11},\ldots, \theta_{jc})$.

i) From the Condition $\boldsymbol{F}$,

(86) $$\sum_{i=1}^{N} \tau_i = \sum_{j=1}^{J} \sum_{c=1}^{C} \xi_{jc} \theta_{jc} G_{jc}(\theta_{jc}).$$

Differentiating (86) with respect to ϕ_{hjc} gives

(87) $$\sum_{i=1}^{N} \tau_{ihc} = \sum_{j=1}^{J} \xi_{jc} \{G_{jc}(\theta_{jc}) + \theta_{jc} G_{hjc}(\theta_{jc})\}$$

where $\tau_{ihc} = \partial \tau_i / \partial \phi_{hjc}$ and $G_{hjc} = (\partial G_{jc}/\partial \theta_{jc})(\partial \theta_{jc}/\partial \phi_{hjc})$.

The payoff for each individual is

(88) $$U_i(\phi_i,\ \phi_{-i}) = \sum_{j=1}^{J} \sum_{c=1}^{C} \xi_{jc} [(\pi_{ijc} - \phi_{ijc}) G_{jc}(\theta_{jc}) + \tau_i(\theta)].$$

Maximizing (88) with repect to ϕ_{ijc} yields

(89) $$\sum_{j=1}^{J} \xi_{jc} [-G_{jc}(\theta_{jc}) + (\pi_{ijc} - \phi_{ijc}) G_{ijc}(\theta_{jc}) + \tau_{ijc}(\theta)] = 0$$

where $G_{ijc} = (\partial G_{jc}/\partial \theta_{jc})(\partial \theta_{jc}/\partial \phi_{ijc})$ and $\tau_{ijc} = \partial \tau_i / \partial \phi_{ijc}$.

The vector $\phi = (\phi_1,\ldots, \phi_N)$ satisfying a system of equations (89) is a Nash equilibrium that is defined as a function of π. Since Condition ***ACR*** requires that $\sum_i \phi_{ijc}(\pi) = \sum_i \pi_{ijc}$ holds for any π, the total over agents of (89) gives

$$(90) \qquad \sum_{i=1}^{N} \tau_{ihc}(\theta) = \sum_{j=1}^{J} \xi_{jc} G_{jc}(\theta_{jc}), \ \forall \theta.$$

While, Condition ***TA*** implies

$$(91) \qquad \tau_{ihc}(\theta) = \tau_{i\rho(i)c}(\rho(\theta)).$$

Furthermore, in light of ***TA***

$$(92) \qquad \sum_{i=1}^{N} \tau_{ihc}(\rho^{-1}(\theta)) = \sum_{i=1}^{N} \tau_{i\rho(i)c}(\rho \cdot \rho^{-1}(\theta)) = \sum_{i=1}^{N} \tau_{i\rho(i)c}(\theta), \ \forall \rho^{-1}, \ \forall \theta$$

where ρ^{-1} is the inverse of ρ. On using (91) and (92)

$$(93) \qquad \sum_{i=1}^{N} \tau_{i\rho(i)c}(\theta) = \sum_{j=1}^{J} \xi_{jc} G_{jc}(\theta_{jc}), \ \forall \rho, \ \forall \phi.$$

Consider the permutation ≯ $\rho(i) = i + k$, $k = 0, 1, \ldots, n-1$ [$\rho(i) = n - i + k$ when $i - k > 1$]. Then equation(93) reads for any $c \in \boldsymbol{C}$,

$$(94) \qquad \begin{aligned} &\tau_{11c} + \tau_{22c} + \cdots + \tau_{NNc} = \sum_{j=1}^{J} \xi_{jc} G_{jc}(\theta_{jc}), \text{ if } k = 0. \\ &\tau_{12c} + \tau_{23c} + \cdots + \tau_{N1c} = \sum_{j=1}^{J} \xi_{jc} G_{jc}(\theta_{jc}), \text{ if } k = 1. \\ &\qquad \cdots\cdots\cdots\cdots \\ &\tau_{1Nc} + \tau_{21c} + \cdots + \tau_{N(N-1)c} = \sum_{j=1}^{J} \xi_{jc} G_{jc}(\theta_{jc}), \text{ if } k = N-1. \end{aligned}$$

Summing all these equations, we obtain

$$(95) \qquad \sum_{i=1}^{N} \sum_{h=1}^{N} \tau_{ihc}(\theta) = N^2 \sum_{j=1}^{J} \xi_{jc} G_{jc}(\theta_{jc})$$

and equation(89) implies

$$(96) \qquad \sum_{i=1}^{N} \sum_{h=1}^{N} \tau_{ihc}(\theta) = N\Big[\sum_{j=1}^{J} \xi_{jc} \{G_{jc}(\theta_{jc}) + \theta_{jc} G_{hjc}(\theta_{jc})\}\Big].$$

Since $\sum_j \xi_{jc} \neq 0$, combining (95) and (96) yields

(97) $$(N-1)G_{jc}(\theta_{jc}) - \theta_{jc}G_{hjc}(\theta_{jc}) = 0.$$

Rearranging the terms and using $\partial\theta_{jc}/\partial\phi_{hjc} = 1$ gives

(98) $$\frac{dG_{jc}(\theta_{jc})/d\theta_{jc}}{G_{jc}(\theta_{jc})} = \frac{N-1}{\theta_{jc}}, \ \forall j \in \boldsymbol{J}, \ \forall c \in \boldsymbol{C}.$$

As in the proof to the **Theorem 6**, we finally have

(99) $$G_{jc}(\theta_{jc}) = \theta_{jc}|\theta_{jc}|^{N-2}, \ \forall j \in \boldsymbol{J}, \ \forall c \in \boldsymbol{C}.$$

ii) In view of (90) and (96), equations(87) and (93) can be written as:

(100) $$T_{ihc} = 0, \ \forall h \in \boldsymbol{N}$$

and

(101) $$T_{i\rho(i)c} = 0, \ \forall \rho$$

where $T_{ihc} = \tau_{ihc} - \tau_{iic}$.

First we show that τ_{ihc} fulfilling (100) and (101) are all zero, that is

(102) $$T_{ihc} = 0, \ \forall i, \ h \in \boldsymbol{N}.$$

By definition

(103) $$T_{ihc} = 0, \ \forall i \in \boldsymbol{N}.$$

Keeping the other terms fixed, consider a permutation ρ which permutates any pair (i, h), we get from (100),

(104) $$T_{ihc} + T_{hic} = 0, \ \forall i, \ h \in \boldsymbol{N}.$$

Let us prove by induction that (101) and (102) imply (103).

Case I: $N = 2$.

Since

(105) $$T_{11c} = T_{22c} = 0$$

we obtain from (100)

(106) $$T_{12c} = T_{21c} = 0.$$

Case II: $N = k$.

Assuming that (100) and (101) imply (102), we verify that this observation holds for $N = k+1$.

Denote

(107) $$s_{ihc} = T_{ihc} + (1+k)T_{k+1,\ hc}.$$

By virtue of (100) and (101) for $N = k+1$, we have for any permutation ρ such that $\rho(N+1) = N+1$:

(108) $$\sum_{i=1}^{k} s_{ihc} = \sum_{i=1}^{k+1} T_{ihc} = 0,\ \forall h = 1,\ldots, k+1$$

and

(109) $$\begin{aligned}\sum_{i=1}^{k} s_{ihc} &= \sum_{i=1}^{k} T_{i\rho(i)c} + (1/k)\sum_{i=1}^{k} T_{k+1,\ \rho(i)c} \\ &= -T_{k+1,\rho(k+1)c} + (1/k)\sum_{i=1}^{k} T_{\rho(i),k+1,c} \\ &= -T_{k+1,c,k+1,c} + (1/k)T_{k+1,c,k+1,c} \\ &= 0.\end{aligned}$$

Hence, by assumption

(110) $$s_{ihc} = 0,\ \forall i,\ h = 1,\ldots, k.$$

Particularly we have

(111) $$s_{hhc} = (1/k)T_{k+1,hc} = 0,\ \forall i,\ h = 1,\ldots, k.$$

and thus

(112) $$T_{ihc} = 0,\ \forall h = 1,\ldots, k+1.$$

In conclusion, equation(100) implies the following:

(113) $$\tau_{i1c} = \tau_{i2c} = \cdots = \tau_{iNc},\ \forall i \in \boldsymbol{N}$$

which means that τ_i is constant as far as $\sum_h \phi_{hic}$ is constant.

Owing to the above arguments, it is only required to verify that Condition $\boldsymbol{N}$ holds for the procedure[(82)-(85)]. Obviously, the family of these processes involves as its subclass a ξMDP Procedure with

$$(114) \qquad \tau_i = \delta_i \sum_{i=1}^{J} \sum_{c=1}^{C} \left(\sum_{j=1}^{J} \phi_{ijc} - \gamma_{jc} \right) Q_{jc}$$

where $\delta_i > 0$ and $\sum_{j=1}^{J} \delta_i = 1$. This ensures Condition $\boldsymbol{N}$ which is included by Condition $\boldsymbol{TN}$. This completes the proof. Q.E.D.

6. FINAL REMARKS

The issue of the present paper has been to design planning procedures which can attain both qualitative and quantitative Pareto optimality. To this end, I have shown the necessary conditions for Pareto optimal product quality in terms of attributes embodied in goods, and named these conditions as the Samuelsonian hedonic conditions. Our procedures are able to achieve Pareto optimal composition of characteristics which satisfy these conditions, so that the original MDP Procedure may be included to be a special one which assumes fixed qualities of goods. I have presented the family of the ζ MDP Procedures that are convergent, neutral, and incentive compatible. It has been easily seen that protagonists in our economy are no longer utilitarian in the orthodox economics, but they are Gorman-Lancasterian-Sen type of individuals who are capable of evaluating goods' quality through their functionings and maximizing their happiness function. Our analysis, however, has been confined to the private goods' quality, and I may present procedures for adjusting quality attributes of public goods in another occasion.

† This is one of a series of papers dedicated to the XXXth Anniversary of the MDP Process. An earlier version of the paper was presented at the autumn meeting of the Japanese Economic Association held at The University of Tokyo, October 17, 1999. Some revisions are made thereafter.

NOTES

1. Gorman's "hidden", but famous classic paper was written in 1956 and finally

published in 1980. To my knowledge, he was the first to use the term, "characterist ics" to represent ingredients of foods. See Deaton and Muellbauer(1980), Gorman and Myles(1987), and Lancaster(1991) for this line of research. A characteristics model is given by Rowcroft(1994). For other interesting approaches to the new consumer theory, see, Sandmo(1973), Jones(1988) and Stokey (1988).

2. In constructing the producers' profit function, I follow Drèze and Hagen(1978, p.510) who wrote that "the implicit price could be computed... and they would in equilibrium be the same for all consumers. So we do not have to make price differentiation among consumers." This fact followed from their assumption of non-singularity of the technological matrix. If the matrix q is non-singular, then its inverse matrix exists, so that p_j can be computed as $\sum_c \pi_{ic} q_{jc}$.

3. In September of 1969 Jacques Drèze and Dominique de la Vallée Poussin, and Edmond Malinvaud presented their papers on quantity-guided planning tâtonnement processes for the optimal provision of public goods at the Brussels Meeting of the Econometric Society. They published their results in 1971 respectively as Malinvaud (1970-1971), and as Drèze and de la Vallée Poussin(1971). Malinvaud noted in his paper that the two approaches closely resembled each other, and among other things, their processes have a remarkable similarity in their adjustment rules for public goods, because each attempted a dynamic representation of Samuelson's conditions for the optimal provision of public goods. Subsequently, Malinvaud published two further papers(1971) and (1972) on the subject. The processes established by three pioneers have become one of the most important contribution in planning theory with public goods, and public economics. They have come to be termed the *MDP Procedure*, which spawned plenty of papers with many fruitful results. The existence of solutions to the MDP Procedure was proved by Claude Henry(1972). See Laffont(1982) and(1985), Mukherji(1990) and Salanié(1998) for lucid summaries of the MDP Procedure. Sato (2000) proposed MDP Procedure where the incentive problem matters for both public and private goods. Also, Sato(2001) annlyzed agents' nonmyopic behaviors in the piecewise linearized MDP Procedure with variable step-sizes.

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