Fairness, Neutrality, and Local Strategy Proofness in Planning Procedures with Public Goods

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Abstract. The advent of the Malinvaud-Drèze-de la Vallée Poussin(MDP) Procedure was epoch making. Malinvaud, Drèze and de la Vallée Poussin sowed the seeds for the subsequent developments in the theory of public goods, and initiated the successful introduction of a game theoretical approach in the planning theory of public goods. Numerous succeeding contributions generated means of providing incentives to correctly reveal preferences for public goods. Nevertheless, the contributions by Malinvaud, Drèze and de la Vallée Poussin share a common weakness. Although the MDP Process possesses desirable properties, they lack incentive compatibility in the strong sense. An attempt to generalize the MDP Process in order to make it robust to preference misrepresentation was first undertaken by Fujigaki and Sato(1981) and characterized in (1982).

The purpose of this paper is three-hold. The first is to clarify a number of versions of neutrality and fairness, so that we clearly distinguish them to obtain results thereon. The second is to investigate the relations among fairness, neutrality, and local strategy proofness, and we summarize the observations in the Table 5.1. The third is to present the family of planning procedures which are simultaneously fair, neutral and locally strategy proof and the family of Generalized Wicksell Procedures is verified to be locally neutral, asymptotically efficient and locally strategy proof.

Key Words : The MDP Procedure, The Generalized MDP Procedure, The Nonlinearized MDP Procedure, The Wicksell Procedure.

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1. INTRODUCTION

1.1. This paper clarifies the relations among fairness, neutrality, and local strategy proofness in the asymptotically efficient planning procedures with public goods. As is well known, local strategy proofness is uniquely and unambiguously defined, while there are a number of versions of fairness and neutrality that should be clearly distinguished to verify the relations among the above three important desiderata, which play significant roles in analyzing continuous planning procedures with public goods.

I show that a shooting range distance of my foregoing analysis is much longer and one can go further than one might have expected, by presenting the family of procedures that are fair, locally neutral, and locally strategy proof, to be lucidly defined below.

Since the seminal works presented by three great pioneers, i.e., Malinvaud(1970–1971), and Drèze and de la Vallée Poussin(1971), no year passed until early nineties without several papers being published in the major journals studying and developing the theory of planning procedures with public goods. It also received much attention in applying the methods for modelling incentives to other areas of scientific research.[1)]

One of the desired properties achievable via planning procedures is *neutrality* advocated by Champsaur(1976), meaning that whatever allocation that is individually rational and Pareto optimal is attainable by the procedures. As for the notions of neutrality, another versions have been subsequently proposed by D'Aspremont and Drèze(1979), Champsaur and Rochet(1983), Laffont and Maskin(1983), and Sato (1983), those of which will be examined below in Section 5.

Another property prerequisite to the right operation of the procedures is called *local strategy proofness*(LSP) according to Champsaur and Rochet(1983), or equivalently, *strongly locally individually incentive compatibility*(SLIIC) in the terminology of Laffont and Maskin(1983), and the former will be used throughout this paper. Under the procedures with this desideratum, every participating agent willingly reveals his/her true willingness to pay, i.e., marginal rate of substitution for public goods as his dominant strategy in a local incentive game he/she plays. This field of research made remarkable progress in the last three decades. Notably, the article of Champsaur and Rochet(1983) further generalized the characterization theorems of Fujigaki and Sato(1981) and (1982), as well as those of Laffont and Maskin(1983), that gave us an impression that this area had already reached an acme.[2)] Much yet remains to be done, for if we were to arrive at a complete theory of planning procedures, we should fully devote ourselves to study their equity and fairness aspects.

Before preceding the next property, it is worthy to point out here that neutrality is related to the second fundamental theorem of welfare economics, so that the planning procedures may well pursue also neutrality as well as local strategy proofness as desiderata. The pursuit of the latter was concluded to impede the former as was shown by Fujigaki and Sato(1981) and (1982) and Sato(2003); that is why we stick to neutrality among other things in this paper. Foregoing attempt was made by Sato(1983) on this issue.

1.2. Malinvaud(1972) was the first economist to devote two sections in his article for discussing equity along planning procedures with public goods. In his procedure which is not necessarily monotonic, he defined his concept of equity which means that the direction of change in utility is the same for every individual taking part in the procedure; namely his procedure treats every agent symmetrically.

Nevertheless, stress has been put on the incentive and efficiency grounds of the procedures, ignoring their *equity* and *fairness* consideractions except for Malinvaud (1972), and Green and Laffont(1979), but Sato(1985) shed new light on this indispensable and unavoidable issue. Having proposed the concepts of equity and fairness in terms of equivalent surplus, he presents a planning procedure satisfying these properties.

Since Foley(1967) put forward the notion of equity, much effort has been devoted to this exciting research field in the last forty years,[3)] bringing about numerous notions of equity and fairness, among which we choose some to be associated with planning procedures.

From the works of Fujigaki and Sato(1981) and (1982), and Sato(2003), we have been aware of the existence of planning procedures which are both asymptotically efficient and locally strategy proof. The thrust of our inquiry in this paper therefore is to verify the compatibility among three desiderata presented above, i.e., fairness, local neutrality, and local strategy proofness in planning procedures with public goods. By fairness we mean equity as well as efficiency. To achieve fairness we propose as a condition imposed on procedures.

Six sections follow. The model is introduced in Section 2, and a review as well as comments on locally strategy proof procedures are given in Section 3. Section 4 presents some definitions of equity and fairness, and we shall show them to be attainable via planning procedures in Section 5, where we also summarize in the table the performance characteristics of the procedures. Conclusion follows in the final section.

2. THE MODEL

2.1. *Notation*

The simplest way to construct a model is to involve only two goods, one of which is taken to be a public good, the other a private good, whose quantities are represented by x and y, respectively.[4] Let there be a production sector and the planning centre which is charged with providing optimal allocation of resources.

(i) *Individual Consumers*

Our society is supposed to contain n individuals. Each consumer $i \in \boldsymbol{N} = \{1, \ldots, n\}$ is characterized by his/her initial endowment of private good $\omega_i > 0$, and his/her utility function u_i.

$$u_i = u_i(x, y_i), \qquad \forall\, i \in N.$$

(ii) *Production Sector*

The production sector is represented by the transformation function $g: \boldsymbol{R}_+ \to \boldsymbol{R}_+$ where $y = g(x)$ signifies the private good quantities needed to produce the public good x. We assume that there is no production of private good, and that the correct information concerning the production technology is known to the planning centre.

2.2. *Assumptions*

Assumption 1. For any $i \in \boldsymbol{N}$

$u(\cdot, \cdot)$ is strictly quasiconcave and at least twice continuously differentiable with

$$u_i^x(x, y_i) = \partial u_i(x, y_i)/\partial x \geqslant 0$$
$$u_i^y(x, y_i) = \partial u_i(x, y_i)/\partial y_i > 0$$

and $\quad u_i^x(x, 0) = 0$ for all $(x, y_i) \geqq 0$.

Assumption 2. g is linear, i.e., $g(x) = x$.

Remark 1. $A2$ is just for the sake of simplicity. Extension to more general convex cases is possible.

2.3. *Definitions*

Following definitions are used throughout this paper.

The marginal rate of substitution between the private good and the public good is denoted by:

$$\pi_i(x, y_i) = \frac{u_i^x(x, y_i)}{u_i^y(x, y_i)}, \qquad \forall i \in \boldsymbol{N}$$

Definition 1. An allocation z is feasible if and only if

$$z \in Z = \left\{(x,\ y_1, \ldots, y_n) \in \boldsymbol{R}_+^{n+1} \mid \sum_i y_i + g(x) = \sum_i \omega_i \right\}.$$

Definition 2. An allocation z is individually rational if and only if

$(\forall i \in \boldsymbol{N})\ [u_i(x, y_i) \geqq u_i(0, \omega_i)]$.

Definition 3. A Pareto optimum for this economy is an allocation $z \in \boldsymbol{N}$ such that, there exists no feasible allocation $\hat{z}$ with:

$(\forall i \in \boldsymbol{N})\ [u_i(\hat{x}, \hat{y}_i) \geqq u_i(x, y_i)]$

$(\exists j \in \boldsymbol{N})\ [u_j(\hat{x}, \hat{y}_j) > u_j(x, y_j)]$.

These assumptions and definitions altogether give us a condition for Pareto optimality in our economy.

Lemma 1. *Under Assumptions 1–2, a necessary and sufficient condition for an allocation $z \in \boldsymbol{Z}$ to be Pareto optimal is*

$$\sum_i \pi_i(x, y_i) \leqq 1 \quad \text{and} \quad x > 0 \quad \Rightarrow \quad \sum_i \pi_i(x, y_i) = 1.$$

Furthermore, conventional mathematical notation will be used throughout in the same manner as in my previous article(1983). Hereafter, all variables are assumed to be functions of time, and an overdot signifies a time derivative of a variable. However, the argument t will often be omitted unless confusion would arise. Given any n-dimensional vector $v = (v_1, \ldots, v_n)$, v_{-1} stands for any $(n-1)$-dimensional vector $(v_1, \ldots, v_{i-1},\ v_{i+1}, \ldots, v_n)$. We shall denote $v = (v_i, v_{-i})$.

3. PLANNING PROCEDURES AS A LOCAL INCENTIVE GAME FORM

3.1. *Description of Procedures*

This section provises a formal presentation of the procedures. A planning procedure is an algorithm, thereby the centre revises the whole allocation according to the information on preferences collected from individuals as well as that on technology of the production sector.

Although the well-known MDP Procedure possesses nice properties, it lacks a strong incentive property. An attempt to generalize the MDP Procedure in order to make it robust to preference misrepresentation on the part of myopic agents was first made by Fujigaki and Sato(1981) who established a family of planning procedures for providing public goods with the following properties:

ⅰ) The procedure converges monotonically to an individually rational Pareto optimum, even if the agents do not reveal their true individual valuation, i.e., marginal rate of substitution(MRS) for a public good.

ⅱ) Revealing his true MRS is always a dominant strategy for each myopically behaving agent, which entails $\delta = 1/n$, $\forall i \in \boldsymbol{N}$.

ⅲ) The procedure generates in the feasible allocation space the same solution path as the original MDP Procedure with equal distribution of the instantaneous surplus generated at each iteration, which leaves no influence of the centre on the final plan. Hence the procedure is non-neutral.

Remark 2. The property ⅱ) is a remarkable one not enjoyed by the original MDP Procedure except when there are only two agents with equal surplus sharing.

There is another case where neutrality property is unattainable. The literature on incentives in planning procedures has commonly supposed myopic behaviour on the part of the agents, which is one of the crucial underpinning for the theoretical development in this field, obtaining numerous desirable properties in connection with the MDP Procedure.[5)] Champsaur and Laroque(1981) and (1982), and Laroque and Rochet(1983), however, analyse how the properties of the MDP Procedure cannot choose but change under non-myopic assumption. Namely, they treat the case where each agent attempts to forecast the influence of his announcement to the planning centre over a time horizon predetermined and optimizes his responses accordingly. They verify that, under suitable assumptions, if this horizon is long enough, any noncooperative equilibrium of intertemporal game achieves an approximately Pareto optimal allocation. In such an equilibrium, however, the control of the centre on the final allocation is negligible. Hence, the procedure becomes non-neutral in this case too.

Their attempt, however, is to bridge the gap between the local instantaneous game and global game, as was pointed out by Hammond(1979). There is another line of research pursued by Truchon(1984) who also studies the non-myopia in the MDP Procedure ; his claim of novelty is to introduce the threshold into the original MDP, to obtain some interesting results, keeping neutrality property preserved.

Beginning our review with a model of a planning procedure defined by the following formulae we denoted as *Process* $P(G,\ T)$:

$$\begin{cases} \dot{x} = G\left(\sum_j s_j - 1\right) \\ \dot{y}_i = -s_i x + T_i\left(s,\ \sum_j s_j - 1\right), \quad \forall\, i \in \boldsymbol{N} \end{cases}$$

where G(•) and T(•, •) are both continuously differentiable, and $s = (s_1,\ \ldots, s_n)$ is a vector of marginal rates of substitution (MRSs) *announced* at date t, which may be different from the true values.

In our works (1981) and (1982), we gave a set of conditions, which we thought, should be satisfied by the planning procedures. We demonstrated that these conditions characterize our procedure: viz, *Process* $P(\tilde{G},\ \tilde{T})$ or the *Generalized MDP Procedure* is characterized as:

$$\begin{cases} \dot{x} = a\left(\sum_j s_j - 1\right)\left|\sum_j s_j - 1\right|^{n-2}, & a \in \boldsymbol{R}_{++} \\ \dot{y}_i = -s_i x + \dfrac{1}{n}\left(\sum_j s_j - 1\right)\dot{x}, & \forall\, i \in \boldsymbol{N}. \end{cases}$$

3.2. *General Characterization of the Family of Locally Strategy Proof Procedures*

The above characterization seems to be complete except for one: that is, the quantity revision of public good is required to be a function of the difference between the sum of the MRSs and the MRT. We knew that this requirement played a crucial role in proving our characterization theorem.

It is therefore quite natural to consider if one could imagine a weaker requirement which gives a general characterization theorem of quantity-guided locally strategy proof procedures that induce each agent to announce his true preferences under the noncooperative myopic behavioural hypothesis. To give a satisfactory answer to this issue, let us describe a generic model of a planning procedure for a public good and a private good as:

$$\begin{cases} \dot{x} = X(s), \\ \dot{y}_i = Y_i(s), \quad \forall i \in \boldsymbol{N}. \end{cases}$$

Champsaur and Rochet(1983) give a systematic study on locally strategy proof procedures, and further generalize our characterization theorems of (1981) and (1982) as well as those of Laffont and Maskin(1983). They show that the class of locally strategy proof procedures is in fact large enough. Now we know that several subclasses belong to this class, viz, the procedures of Bowen, Wicksell, Nonlinearized MDP, Generalized MDP, Green-Laffont, Champsaur-Rochet, Laffont-Maskin, and Generalized Wicksell which will be analyzed below.[6)]

For the sake of completeness we shall briefly resume the resulting theorem on planning procedures which are locally strategy proof and dynamically efficient.

The normative conditions they imposed, which are essentially the same as ours, are in order.

Condition 1. Feasibility or Balancedness

$$(\forall s \in R^n)\left[\sum\nolimits_i Y_i(s) + X(s) = 0\right].$$

Condition 2. Monotonicity or Individual Rationality

$$(\forall s \in \boldsymbol{R}^n)(\forall i \in \boldsymbol{N})\,[s_i X(s) + Y_i(s) \geqq 0].$$

Condition 3. Pareto Efficiency

$$(\forall s \in \boldsymbol{R}^n)\left[X(s) = 0 \quad \Leftrightarrow \quad \sum\nolimits_i s_j = 1\right].$$

Condition 4. Local Strategy Proofness

$$(\forall i \in N)(\forall s \in \boldsymbol{R}^n)(\pi_i \in \boldsymbol{R})$$

$$[\pi_i X(\pi_i,\ s_{-1}) + Y_i(\pi_i,\ s_{-1}) \geqq \pi_i X(s) + Y_i(s)]$$

where $\pi_i = \pi_i(x, y_i)$.

These conditions altogether lead to the characterization theorem. With the view to stating the theorem, we introduce some notations here.

Define the subsets of R^n as follows:

$$\boldsymbol{S}_+ = \left\{s \in \boldsymbol{R}^n \mid \sum\nolimits_i s_i \geqq 1\right\}$$

$$\boldsymbol{S}_- = \left\{ s \in \boldsymbol{R}^n \mid \sum\nolimits_i s_i \leqq 1 \right\}$$

$$\bar{\boldsymbol{S}} = \boldsymbol{S}_+ \cap \boldsymbol{S}_- = \left\{ s \in \boldsymbol{R}^n \mid \sum\nolimits_i s_i = 1 \right\}.$$

Moreover we set for $s \in \boldsymbol{S}_+$

$$\Delta(s) = \{ \mu \in \bar{S} \mid \forall i \in N, \mu_i < s_i \}.$$

Let us call $\mathscr{X}$ the set of functions, defining planning procedures such that

$$X(s) = 0 \text{ for } s \text{ in } \bar{\boldsymbol{S}}$$

$$X(s) > 0 \text{ for } s \text{ in } \boldsymbol{S}_+ \backslash \bar{\boldsymbol{S}}$$

$$X(s) < 0 \text{ for } s \text{ in } \boldsymbol{S}_- \backslash \bar{\boldsymbol{S}}$$

Theorem 1. [Champsaur and Rochet(1983)]

Let $X \in \mathscr{X}$ have at least one $(n-1)$ continuous cross partial derivative on the half space $\boldsymbol{S}_+$. Then, the corresponding mapping $z^x = (x, y_1^x, \ldots, y_n^x)$ is balanced if and only if there exists a continuous function f^+ from $\bar{\boldsymbol{S}}$ to $\boldsymbol{R}_+$, the support of which is equal to $\bar{\boldsymbol{S}}$, such that

$$X(s) = \int_{\Delta(S)} f^+(\mu) d\mu, \quad \forall s \in \boldsymbol{S}_+$$

$$Y_i(s) = -\int_{\Delta(S)} \mu f^+(\mu) d\mu, \quad \forall s \in \boldsymbol{S}_+$$

where $d\mu$ is the Lebesgue measure on $\bar{\boldsymbol{S}}$.

Remark 3. $X(s)$ can be defined on $\bar{\boldsymbol{S}}$ in a similar manner with a function f^-: $\bar{\boldsymbol{S}} \to \boldsymbol{R}_+$. See also Corollary 3 in Champsaur and Rochet(1983) for the mapping z^x which verifies all the Conditions 1–4.

General characterization theorems of locally strategy proof procedures are proved by Laffont and Maskin(1983) under the regularity conditions. Weaker assumptions are sufficient for the theorem proved by Champsaur and Rochet(1983), who relax the differentiability of decision functions which define planning procedures. Besides this rather mathematical work, Sato(1983) should be mentioned, which establishes that the Generalized MDP Procedure proposed by Fujigaki and Sato(1981) and (1982) is more robust than we have expected: namely, its efficient operation does not even require correct revelation for each individual to be a Nash equilibrium strategy shown by Roberts(1976), but needs only aggregate correct revelation to be a Nash equilibrium. Moreover he showed that the public good decision function could be characterized as the same form with two additional conditions without resorting to Condition 4. See Sato(1983) for the details.

4. EQUITY AND FAIRNESS

Since Foley(1967) took equity into consideration, at least three concepts have been proposed in the context of planning procedures. We shall therefore consider the following notions in this section: (i) Foley's no envy equity(1967), (ii) Malinvaud's equity(1972), and (iii) Green and Laffont's equity(1979), those of which we shall examine in this section.

(i) *Foley's Equity* About the last four decades passed since Foley devoted one section to the concept of equity, where he advocated equity based on envy. With our notation, we have

Definition 4. An allocation z is *Foley equitable if*

$$(\forall i, j \in \boldsymbol{N})\,[\,u_i(x, y_i) \geqq u_i(x, y_j)\,].$$

We say that an allocation is *no envy equitable* if nobody prefers the other person's bundle of goods to his own, i. e., every agent subjectively feels himself to be better off than anyone of the others. Moreover, we say that an allocation is *Foley fair* if it is both Foley equitable and Pareto efficient. The Foley's theorem, proved by Kolm(1972) and Varian(1974), states that under suitable assumptions there exists a Foley fair allocation in a pure exchange economy. In Suzumura and Sato(1985), we conclude, by presenting several counterexamples, that none of the celebrated solutions except the public competitive equilibrium can attain this Foley fairness in our model economies with public goods, where existence of fair allocations is assured.

How about the MDP Procedure then? The answer is in the affirmative, since it possesses neutrality put forward by Champsaur(1976), in the sense that a planning procedure is neutral if it can attain whatever allocation which is individually rational and Pareto optimal. Hence the MDP Procedure can achieve as one of its equilibrium points a fair allocation which is individually rational. In so doing, however, the centre must acquire a priori the relevant information before the beginning of the procedure's working so as to reach the fair allocation. For this purpose, one has to collect the necessary but privately held information from individual agents, in order to establish the fair allocation. Neutrality will be considered in detail in the next section.

(ii) *Malinvaud's equity*

As was already mentioned in the introduction, Malinvaud(1972) took precedence over all others in accounting the equity issue remarkably important in the theory of planning procedures. His idea is to treat individual agents symmetrically along the operation of the procedure.

We can describe Malinvaud's equity in a somewhat different way from the original expression. Let $\dot{u}_i(x, y_i) = u_y(x, y_i)(\pi_i X + Y_i)$.

Definition 5. An allocation z is *Malinvaud equitable if*

$$(\forall i, j \in \boldsymbol{N})\ [\,\dot{u}_i(x, y_i)\,\dot{u}_j(x, y_j) \geqq 0\,].$$

This amounts to say that the direction of the utility change is the same over individuals, namely, if instantaneous utility of one person increases(decreases), that of the other also increases(decreases). This concept is formulated in a continuous framework, which hints another condition to be considerable in planning processes.

Before rushing to the next concept, we would like to comment on the Malinvaud's equity. His earlier concept was that a procedure is said to be equitable if every participant could benefit from the revision of allocation. This notion, however, substantially equivalent to what we now call monotonicity, that is to say, the monotonic procedures satisfy Malinvaud's earlier notion included in the definition 5, involving also negative monotonicity. Additionally, Malinvaud(1971) already pointed out the issue of equity, and compared quantity and price guided procedures from the incentive viewpoint.[7)]

(iii) *Green and Laffont's equity*

In their celebrated book(1979, p.274), they wrote, "... By choosing equal shares of the cost, the procedure can be made equitable in the following sense: if the agents consider the procedure before knowing their own preferences, in the spirit of the

Rawlsian approach, no particular agent is favored." According to their criterion, the procedures of Bowen, Green and Laffont, and Wicksell can be equitable for the agents as in the original position à la Rawls, since these procedures can involve equal shares of the cost.

5. FAIRNESS, NEUTRALITY AND LOCAL STRATEGY PROOFNESS

5.1. *Champsaur Neutrality*

It was Champsaur(1976) who introduced neutrality as a desideratum that should be achieved via planning procedures. It says that any individually rational Pareto optimum can be attained by the procedures.

Originally, the problem that Champsaur considered was whether a procedure can reach all the elements of the set $\boldsymbol{P}$ of all individually rational Pareto efficient programmes. If it is true, we say that the procedure is neutral, which reads in the mathematical expression:

Condition 5. Champsaur Neutrality

$$(\forall \bar{z} \in \boldsymbol{P})(\exists \delta \in \boldsymbol{S})(\exists z(\cdot) \in F(\delta))[\bar{z} = \lim_{t \to \infty} z(t)]$$

where $\delta = (\delta_1, \ldots, \delta_n)$ is a deterministic parameter in the simplex

$$S = \left\{(\delta_1, \ldots, \delta_n) \in \boldsymbol{R}_+^n \mid \sum_i \delta_i = 1\right\}$$

and $F(\delta)$ is the set of solutions.

Champsaur(1976) established the theorem that the MDP Procedure is neutral in the sense that $z(\infty, \delta)$ describes all the elements of the set $\boldsymbol{P}$ according to δ in the simplex $\boldsymbol{S}$. Subsequently, Cornet and Lasry(1976), Cornet(1977a, b, c) and (1983) further generalized the results of Champsaur by dropping the uniqueness of solutions and strict concavity of utility functions. Furthermore, it is worth pointing that the neutrality of the procedures signifies that the planning centre can accomplish whatever programmes belonging to $\boldsymbol{P}$, in accordance with the distributional parameter vector selected by the centre before the beginning of the procedure. There is hence a large freedom left to the centre concerning a compatibility between distributional equity and allocative efficiency, and, not to mention, Green and Laffont's equity is fulfilled as well.

5.2. *The Effect of Manipulation on Neutrality*

Incidentally, neutrality does not require truthful revelation of preferences as was already verified by Drèze and de la Vallée Poussin(1971), which is not necessarily required to be a dominant strategy. With non-cooperative Nash type manipulation on the part of agents, the original MDP Procedure cannot choose but be modified, as shown by Roberts(1979) as follows:

$$\begin{cases} \dot{x} = (1/(n-1))\left(\sum_j s_j - 1\right) \\ \dot{y}_i = \left\{-s_i + \sigma_i\left(\sum_j s_j - 1\right)\right\}\dot{x}, \quad \forall i \in \boldsymbol{N} \end{cases}$$

where $\sigma_i = (1-\delta_i)/(n-1), \quad \forall i \in \boldsymbol{N}$.

One can easily see that the possible range of σ_i reduces as the number of agents n becomes large. Since

$$0 < \sigma_i < 1/(n-1)$$

and the ranges of σ_i and δ_i coincide only when $n = 2$. Hence, the centre's controllability on the division of social surplus greatly diminishes under manipulation à la Nash, which entails the unattainableness of neutrality, and only the subclass of individually rational Pareto optima is achievable.

In the MDP-type procedures with distributional parameter vector $\delta = (\delta_1, \ldots, \delta_n)$, truthful revelation of preferences for public goods ensues monotonicity and neutrality, albeit it is not a dominant strategy for each player. This implies that true representation of preferences assures, via neutrality, the fulfillment of all of the versions of equity presented in the preceding section. When we argue neutrality, we have to carefully distinguish a number of its versions, and we can say that there are two possibilities as to the relations between neutrality and local strategy proofness. The first is that the latter can prevent the achievement of the former in the original Champsaur's sense, as was shown by Fujigaki and Sato(1981), and the second is that the latter is compatible with the former in the sense of Sato(1983), and Champsaur and Rochet(1983).

5.3. *Another Versions of Neutrality*

As is easily seen from our paper(1981), the Generalized MDP Procedure is not neutral, which means that local strategy proofness could impede the attainment of neutrality. Hence, Sato(1983) proposed another version of neutrality, keeping the same public good decision function as with Condition 4. In mathematical terms it can be stated as follows:

*Condition 6.　Neutrality**

$$(\forall \bar{z} \in \boldsymbol{P})(\exists T \in \mathscr{T})(\exists z(\cdot) \in F(\delta))[\bar{z} = \lim_{t \to \infty} z(t)]$$

where $\mathscr{T}$ is the class of $T = \{T_1,\ldots, T_n\}$ and $z(\cdot)$ is a solution of $P(G, T)$.

With this condition, we could revive neutrality by weakening the strength of incentives from dominant to aggregate Nash. With additional conditions, Sato(1983) could characterize the public good decision function to be the same form as the one obtained with Condition 4.

An extention of this concept was subsequently made by Champsaur and Rochet(1983), who study the neutrality property of the planning procedures and distinguish two notions of neutrality, i.e., local neutrality and global neutrality, which will be defined in order.

Condition 7.　Local Neutrality

A family $\mathscr{X}$ of planning procedures is locally neutral if

$$(\forall s \in \boldsymbol{R}^n)(\forall \delta \in \boldsymbol{S})(\exists X \in \mathscr{X})(\forall i \in \boldsymbol{N})$$

$$\left[\dot{u}_i^X(s) = \delta_i\, \dot{u}^X(s) = \delta_i\Big(\sum\nolimits_j s_j - 1\Big)X(s)\right]$$

where $\dot{u}^X(s) = \dot{u}^X(X(s), Y_i(s)) = u_y(\pi_i X(s) + Y_i(s))$.

Condition 8.　Global Neutrality

A family $\mathscr{X}$ of planning procedures is globally neutral if

$$(\forall \bar{z} \in \boldsymbol{P})(\exists X \in \mathscr{X})(\forall i \in \boldsymbol{N})$$

$$[u_i(x, y_i) > u_i(0, \omega_i)\ \&\ \bar{z} = z(\infty, z^X)].$$

In effect, Champsaur and Rochet(1983) showed that the class of planning procedures fulfilling all of the Conditions 1–4, i.e., the class of procedures which are asymptotically efficient as well as locally strategy proof is in fact large enough. To put it differently, much yet remains to be done by the centre to exert a great influence on the final outcome to be reached by the procedure. Then contrasting result different from ours in (1981) and (1982) stems from the fact that in their process they make the centre adjust instantaneously the decision function $X(s)$ at each iteration t, which entails the simultaneous achievement of local neutrality and local strategy proofness. Whereas, in our Generalized MDP Procedure we proposed in (1981) and (1982), we made the centre preserve the functions throughout the operation of the procedure. Laffont and Maskin(1983) considered the case where the centre can change the mapping z^X itself over time.

Finally, Champsaur and Rochet(1983) went even further and proved that the failure of existence of globally neutral as well as locally strategy proof procedures is general in a large class of economy with more than two agents. See their Proposition 5.

Remark 4. Laffont(1982),(1985b),(1987), and Laffont and Maskin(1983) elaborately examine the locally strategy proof procedures with only two agents. A graphical presentation of original Champsaur neutrality is given in Figure of Laffont(1982) and that of local neutrality in a Figure of Laffont(1985b, p.21).

5.4. *The Family of Generalized Wicksell Procedures*

Recall that Champsaur and Rochet(1983) interpret the class of procedures they call the family of Generalized Wicksell Procedures as a subclass belonging to the family of locally strategy proof procedures[8]: for any $s \in \boldsymbol{S}$

$$\begin{cases} X(s) = \alpha > 0 & \text{if } s_i > \mu_i \quad \text{for all } i \\ Y_i(s) = \mu_i \alpha & \text{for all } i \quad \text{if } X(s) = \alpha \\ X(s) = 0 & \text{if } s_i \leqq \mu_i \quad \text{for some } i, \\ Y_i(s) = 0 & \text{for all } i \quad \text{if } X(s) = 0 \end{cases}$$

where $\mu_i = s_i - \delta_i\left(\sum_j s_j - 1\right)$ is an individualized implicit unit tax.

With these definitions we now have the following result.

Theorem 2. [Rochet(1982)]
The family of Generalized Wicksell Procedures defined as above is locally neutral.

As verified by Champsaur and Rochet(1983), the class of locally strategy proof procedures is rich enough, containing at least as its members the procedures of Bowen, two-person MDP, Green-Laffont, Wicksell, Generalized MDP, and Generalized Wicksell, performance of which will be summarized below in the Table 5.1.

Among three alternatives of equity concepts, we select Malinvand equity as a condition which can bring us some fruitful results surrounding the relationship between fairness and some versions of neutrailty.[9)]

Here we show the results in Theorem 2 and Corollary 3 in Champsaur and Rochet(1983).

5.5. *The Class and the Classification of Procedures*

Let $\mathscr{S}$ be the Borel σ-algebra of $\bar{\boldsymbol{S}}$. A locally finite measure μ on $(\bar{\boldsymbol{S}}, \mathscr{S})$ is said to be strictly positive if it is positive, and if its support is equal to $\bar{\boldsymbol{S}}$.

All the above results altogether give us the following theorems.

Theorem 3. [Champsaur and Rochet(1983)]
Let μ^+ be a locally finite and strictly positive measure on $(\bar{S}, \mathscr{S})$. Let $X : \bar{\boldsymbol{S}} \rightarrow R_+$ be defined on S_+ by

$$X(s) = \mu^+(\Delta(s))$$

where $\Delta(s) = \{\mu \in \bar{\boldsymbol{S}} \mid \mu_i < s_i,\ \forall i \in \boldsymbol{N}\}$ then the mapping z^X verifies all the Conditions C1-C4, and C7.

Theorem 4. *The Generalized MDP Procedures achieve all the alove conditions including Malinvand Equity.*

Proof : We know from Theorem in Sato(2003) that there exists a class of procedures which satisfy all conditions, to which the Generalized MDP Procedures belong. A fulfillment of Malinvand Equity immediately follows. II

Let me classify the Family of procedures.

Table 5.1 Performance Characteristics of Planning Procedures

Procedures	Features 1	2	3	4	5	6	7	8
Ⅰ **Two-person case**								
i) *MDP Procedure*	†	†	†	†[a]	†	†	†	†
[Drèze and de la Vallée Poussin(1971)]								
ii) *Bowen Procedure*	†		†	†				†

Procedures	**Features** 1	2	3	4	5	6	7	8
[Green and Laffont(1979)]								
iii) *Green–Laffont Procedure*	†	†[b]	†[b]	†[b]				†[e]
[Green and Laffont(1979)]								
iv) *Nonlinearized MDP Procedure*[10)]	†	†	†	†[a]	†	†	†	†
[Fujigaki and Sato(1981)]								
v) *Laffont–Maskin Procedure*	†	†	†	†	†	†	†	†
[Laffont and Maskin(1983)]								
vi) *Champsaur–Rochet Procedure*	†	†	†	†	†	†	†	†
[Champsaur and Rochet(1983)]								
vii) *Generalized Wicksell Procedure*	†	†	†	†	†	†	†	†
[Champsaur and Rochet(1983)]								
viii) *Wicksell Procedure*	†	†	†[c]	†	†			†
ix) *Generalized MDP Procedure* [Sato(2003)]	†	†	†	†	†	†[f]	†	†
Ⅱ **Many–person case**								
i) *MDP Procedure*	†	†	†		†	†	†	†
ii) *Bowen Procedure*	†		†[d]	†				
iii) *Green–Laffont Procedure*	†	†[b]	†[b]	†[b]				†[e]
iv) *Nonlinearized MDP Procedure*	†	†	†	†	†		†	†
v) *Laffont–Maskin Procedure*	†	†	†	†	†	†	†	†
vi) *Champsaur–Rochet Procedure*	†	†	†	†	†	†	†	†
vii) *Generalized Wicksell Procedure*	†	†	†	†	†	†	†	
viii) *Wicksell Procedure*	†	†	†[c]	†	†			†
ix) *Generalized MDP Procedure*	†	†	†	†	†[f]	†	†	†

† means the statement already proven, or a conjecture not yet formally verified, but we consider it to be easily shown.

Key to Features:

1. Feasibility or Balancedness
2. Monotonicity or Individual Rationality
3. Asymptotic or Dynamical Efficiency
4. Local Strategy Proofness

5. Champsaur Neutrality
6. Local Neutrality
7. Global Neutrality
8. Malinvand Equity

ａ. In two-person case, the original MDP Procedure with equal sharing of surplus and the Nonlinearized MDP coincide.

ｂ. With the aid of a critical assumption, Green and Laffont(1979) verified that the features 3 and 4 could be satisfied by their procedure. Furthermore, their procedure could attain whatever Pareto optimum via the choice of sharing cost, which plays a similar role as distributional parameter vector in the MDP-type procedures. Feature 2 holds only for pivotal agents. See Green and Laffont(1979, Ch. 16).

ｃ. For this point, see Laffont and Rochet(1985, p. 316).

ｄ. The Bowen Procedure does not generally stop at a Pareto optimum unless a special assumption that the median of the MRSs coincide with their mean, albeit the earliest procedure designed to be locally strategy proof. See, Green and Laffont(1979, Ch. 14).

ｅ. Of course, the Green-Laffont Procedure satisfy Green-Laffont Equity.

ｆ. The Generalized MDP Procedure also fulfills Neutrality.*

Remark 5. With many public goods there can arrive several difficulties with some of the above procedures. The MDP Procedure can treat any number of public goods, whereas the Generalized MDP needs an additional assumption to avoid the boundary problem, i.e., the boundary condition $\sum_i \pi_i(x(0), y(0)) > 1$, which the Generalized Wicksell Procedure can dispense with, since it does not impose the differentiability of the public good decision function. Next, without separability of preferences, the Green-Laffont Procedure may cycle and it is not stable when there are many public goods, the graphical illustration is presented by Green and Laffont(1979, p. 278, Fig. 16. 1). Sato(1998) and (2003) dealt with the related problems.

6. CONCLUDIND REMARKS

Since the contributions of Malinvaud(1970-71), and Drèze and de la Vallée

Poussin(1971), numerous advances have been made in the design and analyses of planning procedures thereby the allocation of resources is organized in a way which suits some criterion, and for that purpose, information must be elicited from decentralized agents. The properties of the procedures have been fully developed; especially, we are now well acquainted with neutrality and local strategy proofness. Additional criteria have been considered in this paper, viz, equity and fairness, but not thoroughly, though.

Almost all of the literature uses as a theoretical foundation the MDP Procedure, whose properties including neutrality and local strategy proofness have been clearly observed, hence, one can legitimately wonder if it is worth to take seriously enough to examine equity and fairness of the family of MDP Procedures. Of course we have discovered another type of procedures which are locally strategy proof. Thus, it is to be hoped that these procedures should have equity and fairness properties, which make them more attractive and powerful.

The purpose of this paper has been three-hold. The first is to clarify a number of versions of neutrality and fairness, so that we have clearly distinguished them to obtain results thereon. The second is to investigate the relations among fairness, neutrality, and local strategy proofness, and we have summarized the observations in the Table 5.1. The third is to present the family of planning procedures which are simultaneously fair, neutral and locally strategy proof, i.e., the family of Generalized Wicksell Procedures has been verified to be locally neutral, asymptotically efficient and locally strategy proof.

† This is one of the series of papers dedicated to the XXXth annuversary of the MDP Procedure. An earlier version of this paper was presented at the annual meeting of the Japan Association of Theoretical Economics and Econometrics held at Nagoya University, November 3, 1986. The author is grateful to Professor Toshihiro Sato for his incisive comments.

NOTES

1. The relevant references are, among others, Laffont(1985a, Ch.10), Laffont and Maskin(1982), and Lollivier and Rochet(1983).

2. Laffont(1985b) gave a systematic as well as pedagogical survey of numerous

fascinating results on incentive compatibility in the generic context of planning procedures, but he does not treat their equity aspects. See also Roberts(1986) for another challenging issue: that is, he tries to relax simultaneously both the assumptions of myopia and of complete information in an iterative planning framework, in which the agents are initially imperfectly informed about each other, so they learn about one another to predict future behaviour of the others. Furthermore, Laffont and Rochet(1985) established the duality between locally strategy proof quantity-guided planning procedures and nonlinear price planning procedures, which could overturn the widely held view that the quantity-guided planning procedures are concluded to excel the price-guided ones in incentive property. In this respect, see Malinvaud(1971, p.92), and Sato(1986). Rochet(1982) brought together some developments in this field of reseach.

3. Thomson and Varian(1985) provided a comprehensive and wide ranging description and analysis of equity based on envy.

4. Generalization to multidimensional space of goods is straightforward by employing the method developed by Sato(2003).

5. See, for example, Laffont(1985b, p.14) and Sato(2001) for a justification of myopia.

6. It was the Green-Laffont Procedure(1979) that was proposed as an earliest member belonging to the family of locally strategy proof procedures, nonetheless, they needed a special assumption which prompted critical comments.

7. See Sato(1986) for this issme.

8. They consider a Dirac measure at point μ of S. For a simple Wicksell Procedure with *ex ante* deterministic sharing of production costs, see Laffont and Rochet(1985, p.315).

9. The discrete version of MDP Procedure proposed by Champsaur, Drèze and Henry(1977) can be easily shown to achieve some notions of neutrality, which entails

a fulfillment of the equity concepts presented above.

10. Sato(2003) ranamed the Fujigaki-Sato Procedure as the "Nonlinearized MDP Procedure" and then presented the genuine "Generalized MDP Procedure" which fulfills all the features enumerated here.

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